

## GENERALIZED STRUCTURED COMPONENT ANALYSIS

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We propose an alternative method to partial least squares for path analysis with components, called generalized structured component analysis. The proposed method replaces factors by exact linear combinations of observed variables. It employs a well-defined least squares criterion to estimate model parameters. As a result, the proposed method avoids the principal limitation of partial least squares (i.e., the lack of a global optimization procedure) while fully retaining all the advantages of partial least squares (e.g., less restricted distributional assumptions and no improper solutions). The method is also versatile enough to capture complex relationships among variables, including higher-order components and multi-group comparisons. A straightforward estimation algorithm is developed to minimize the criterion.

Key words: Path analysis with components, partial least squares, alternating least squares.

### 1. Introduction

Partial least squares (Wold, 1966, 1973, 1982) is employed for path analysis with components or weighted composites of observed variables. Partial least squares (PLS) estimates model parameters by the so-called fixed point (FP) algorithm (Lyttekens, 1968, 1973; Wold, 1965, 1981). In the FP algorithm, a set of model parameters is divided into subsets, and each subset is “partially” estimated by ordinary least squares (OLS) with other subsets fixed. This OLS estimation is cycled through repeatedly until convergence is reached.

PLS avoids improper solutions (e.g., factor correlation estimates greater than  $\pm 1$ , negative variance estimates, and so on) since it replaces factors by linear composites of observed variables as in component analysis (Meredith & Millsap, 1985; Schönemann & Steiger, 1976). PLS provides unique component score estimates of cases, which may be used for additional analyses or for selection or segmentation of the cases. Furthermore, since its parameter estimation is based on (partial) OLS, PLS does not rely on stringent distributional assumptions, such as the multivariate normality of observed variables, which is often violated in nonexperimental data (Micceri, 1989). Nonetheless, PLS does not solve a global optimization problem for parameter estimation, indicating that there exists no single criterion consistently minimized or maximized to determine model parameter estimates (Jöreskog & Wold, 1982; Fornell & Bookstein, 1982). Due to the lack of a global optimization criterion, it is difficult to evaluate the PLS procedures (McDonald, 1996). Also, it is not guaranteed that the obtained PLS solutions are optimal in a well-defined sense (Coolen & de Leeuw, 1987). More seriously, PLS provides no mechanism to evaluate the overall goodness of fit of the model. Given no overall goodness of fit measures, it

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is difficult to examine how well the model fits to the data and also to compare it with alternative models. Therefore, it seems to be the major limitation of PLS that no global optimization criterion is available, although there were a few attempts to alleviate the problem (e.g., Bookstein, 1982; Hanafi & Qannari, 2002).

In this paper, we propose a new method, called generalized structured component analysis (GSCA) that avoids the major drawback of PLS. As the name suggests, GSCA lies in the tradition of component analysis. It substitutes components for factors as in PLS. Unlike PLS, however, GSCA offers a global least squares optimization criterion, which is consistently minimized to obtain the estimates of model parameters. GSCA is thus equipped with an overall measure of model fit while fully maintaining all the advantages of PLS (e.g., less restricted distributional assumptions, no improper solutions, and unique component score estimates). In addition, GSCA handles more diverse path analyses, compared to PLS. For example, it can easily fit the same path model to more than one group with optional impositions of across-group constraints, which PLS cannot accommodate.

The remaining sections of this paper are organized as follows. In section 2, we shall discuss GSCA in detail. We present the model for GSCA, the estimation procedure of model parameters, and simple extensions of the model. In section 3, we shall investigate the recovery of parameters in GSCA through a Monte-Carlo simulation study. In section 4, we shall illustrate the empirical validity of GSCA with an example, compared with PLS. The final section briefly summarizes the previous sections and discusses further prospects for GSCA.

## 2. The Method

### 2.1. The Model

Let  $\mathbf{Z}$  denote an  $N$  by  $J$  matrix of observed variables. Assume that  $\mathbf{Z}$  is columnwise-centered and scaled to unit variance. Then, the model for GSCA may be expressed as

$$\begin{aligned}\mathbf{ZV} &= \mathbf{ZWA} + \mathbf{E}, \\ \mathbf{\Psi} &= \mathbf{\Gamma A} + \mathbf{E},\end{aligned}\tag{1}$$

where  $\mathbf{\Psi} = \mathbf{ZV}$ , and  $\mathbf{\Gamma} = \mathbf{ZW}$ . In (1),  $\mathbf{\Psi}$  is an  $N$  by  $T$  matrix of all endogenous observed and composite variables,  $\mathbf{\Gamma}$  is an  $N$  by  $D$  matrix of all exogenous observed and composite variables,  $\mathbf{V}$  is a  $J$  by  $T$  matrix of component weights associated with the endogenous variables,  $\mathbf{W}$  is a  $J$  by  $D$  matrix of component weights for the exogenous variables,  $\mathbf{A}$  is a  $D$  by  $T$  supermatrix consisting of a matrix of component loadings relating components to their observed variables, denoted by  $\mathbf{C}$ , in addition to a matrix of path coefficients between components, denoted by  $\mathbf{B}$ , that is,  $\mathbf{A} = [\mathbf{C}, \mathbf{B}]$ , and  $\mathbf{E}$  is a matrix of residuals.

To illustrate (1), we present two kinds of exemplary relationships among variables. The first one is displayed in Figure 1. In Figure 1, square boxes are used to indicate observed variables ( $\mathbf{z}_i$ ,  $i = 1, \dots, 8$ ), circles are used to represent components ( $\boldsymbol{\gamma}_1$  and  $\boldsymbol{\gamma}_2$ ) or residuals ( $\mathbf{e}_i$  and  $\mathbf{d}$ ), and straight arrows are used to signify that the variable at the base of an arrow affects the variable at the head of the arrow. The figure indicates that each of two components is a linear combination or a weighted composite of four observed variables, that is,  $\boldsymbol{\gamma}_1 = \sum_{i=1}^4 \mathbf{z}_i w_i$ , and  $\boldsymbol{\gamma}_2 = \sum_{i=5}^8 \mathbf{z}_i w_i$ , where  $w_i$  is a component weight. The components are specified to affect the observed variables, that is,  $\mathbf{z}_i = \boldsymbol{\gamma}_1 c_i + \mathbf{e}_i$  if  $i \leq 4$ , and  $\mathbf{z}_i = \boldsymbol{\gamma}_2 c_i + \mathbf{e}_i$  otherwise, where  $c_i$  is a component loading. It indicates that all observed variables can be regarded as reflective indicators in a sense similar to PLS since they are influenced by their components rather than factors (Fornell & Cha, 1994). It is also found that  $\boldsymbol{\gamma}_1$  affects  $\boldsymbol{\gamma}_2$ , that is,  $\boldsymbol{\gamma}_2 = \boldsymbol{\gamma}_1 b + \mathbf{d}$ , where  $b$  is a path coefficient. Let

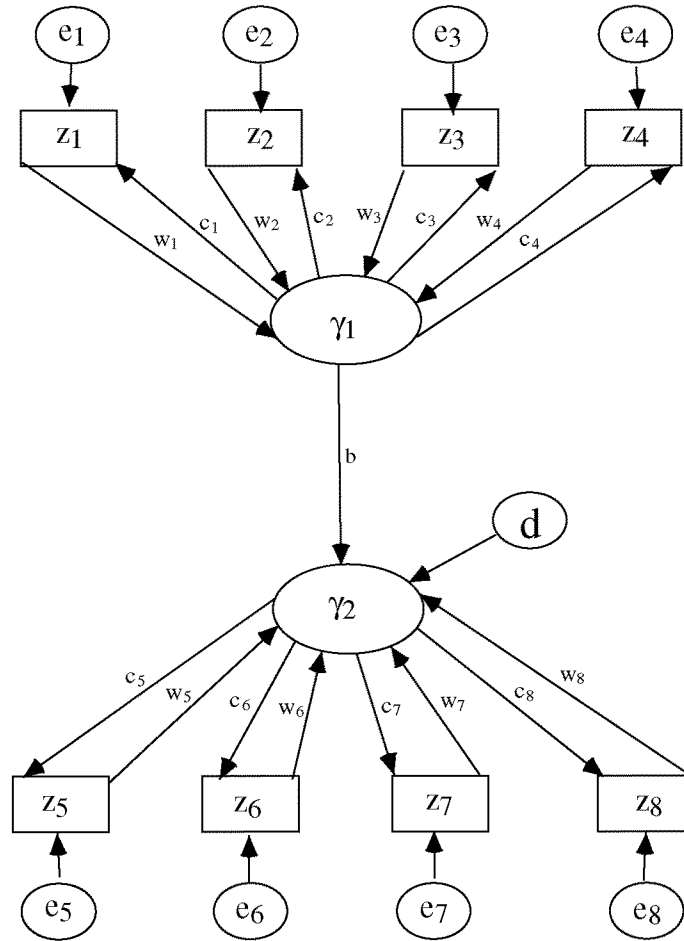


FIGURE 1.  
A path model with two components.

$$\mathbf{Z} = [z_1, z_2, z_3, z_4, z_5, z_6, z_7, z_8],$$

and

$$\mathbf{E} = [e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, d].$$

This relationship can then be expressed as

$$\mathbf{Z} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & w_5 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & w_6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & w_7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & w_8 \end{bmatrix}$$

$$= \mathbf{Z} \begin{bmatrix} w_1 & 0 \\ w_2 & 0 \\ w_3 & 0 \\ w_4 & 0 \\ 0 & w_5 \\ 0 & w_6 \\ 0 & w_7 \\ 0 & w_8 \end{bmatrix} \begin{bmatrix} c_1 & c_2 & c_3 & c_4 & 0 & 0 & 0 & 0 & b \\ 0 & 0 & 0 & 0 & c_5 & c_6 & c_7 & c_8 & 0 \end{bmatrix} + \mathbf{E},$$

$$\mathbf{ZV} = \mathbf{ZWA} + \mathbf{E},$$

$$\mathbf{\Psi} = \mathbf{\Gamma A} + \mathbf{E}. \quad (2)$$

In (2),  $\mathbf{\Psi} = [\mathbf{Z}, \boldsymbol{\gamma}_2]$ ,  $\mathbf{\Gamma} = [\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2]$ , and  $\mathbf{A} = [\mathbf{C}, \mathbf{b}]$ , where

$$\mathbf{C} = \begin{bmatrix} c_1 & c_2 & c_3 & c_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_5 & c_6 & c_7 & c_8 \end{bmatrix},$$

and

$$\mathbf{b} = \begin{bmatrix} b \\ 0 \end{bmatrix}.$$

We contemplate another, more complicated relationship among variables, presented in Figure 2. In Figure 2, four components ( $\boldsymbol{\gamma}_1$ ,  $\boldsymbol{\gamma}_2$ ,  $\boldsymbol{\gamma}_3$ , and  $\boldsymbol{\gamma}_4$ ) are specified, each of which is obtained as a linear combination of three observed variables. Among them,  $\boldsymbol{\gamma}_1$  and  $\boldsymbol{\gamma}_2$  do not affect the observed variables, so that no loadings are involved. Thus,  $\boldsymbol{\gamma}_1$  and  $\boldsymbol{\gamma}_2$  are associated with forma-

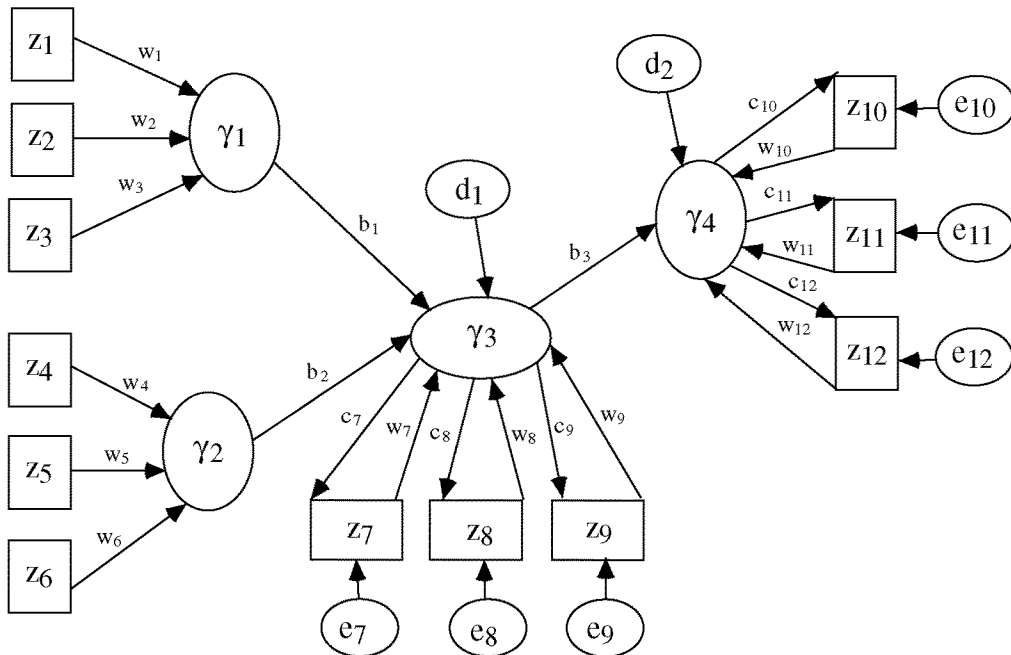


FIGURE 2.  
A path model with four components.

tive indicators since they are simply formed by their observed variables (Fornell & Cha, 1994). On the other hand,  $\gamma_3$  and  $\gamma_4$  are associated with reflective indicators as in the previous example. Moreover, both  $\gamma_1$  and  $\gamma_2$  affect  $\gamma_3$ , which influences  $\gamma_4$ . Let

$$\mathbf{Z} = [\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3, \mathbf{z}_4, \mathbf{z}_5, \mathbf{z}_6, \mathbf{z}_7, \mathbf{z}_8, \mathbf{z}_9, \mathbf{z}_{10}, \mathbf{z}_{11}, \mathbf{z}_{12}],$$

and

$$\mathbf{E} = [\mathbf{e}_7, \mathbf{e}_8, \mathbf{e}_9, \mathbf{e}_{10}, \mathbf{e}_{11}, \mathbf{e}_{12}, \mathbf{d}_1, \mathbf{d}_2].$$

Then, the relationship in Figure 2 can be expressed as

$$\mathbf{Z} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & w_7 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & w_8 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & w_9 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & w_{10} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & w_{11} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & w_{12} \end{bmatrix} = \mathbf{Z} \begin{bmatrix} w_1 & 0 & 0 & 0 \\ w_2 & 0 & 0 & 0 \\ w_3 & 0 & 0 & 0 \\ 0 & w_4 & 0 & 0 \\ 0 & w_5 & 0 & 0 \\ 0 & w_6 & 0 & 0 \\ 0 & 0 & w_7 & 0 \\ 0 & 0 & w_8 & 0 \\ 0 & 0 & w_9 & 0 \\ 0 & 0 & 0 & w_{10} \\ 0 & 0 & 0 & w_{11} \\ 0 & 0 & 0 & w_{12} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & b_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & b_2 & 0 \\ c_7 & c_8 & c_9 & 0 & 0 & 0 & 0 & b_3 \\ 0 & 0 & 0 & c_{10} & c_{11} & c_{12} & 0 & 0 \end{bmatrix} + \mathbf{E},$$

$$\mathbf{ZV} = \mathbf{ZWA} + \mathbf{E},$$

$$\mathbf{\Psi} = \mathbf{\Gamma A} + \mathbf{E}.$$

(3)

In (3),  $\mathbf{\Psi} = [\mathbf{z}_7, \mathbf{z}_8, \mathbf{z}_9, \mathbf{z}_{10}, \mathbf{z}_{11}, \mathbf{z}_{12}, \gamma_3, \gamma_4]$ ,  $\mathbf{\Gamma} = [\gamma_1, \gamma_2, \gamma_3, \gamma_4]$ , and  $\mathbf{A} = [\mathbf{C}, \mathbf{B}]$ , where

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ c_7 & c_8 & c_9 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{10} & c_{11} & c_{12} \end{bmatrix}, \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} b_1 & 0 \\ b_2 & 0 \\ 0 & b_3 \\ 0 & 0 \end{bmatrix}.$$

As shown in the examples,  $\mathbf{V}$ ,  $\mathbf{W}$ , and  $\mathbf{A}$  in (1) are structured according to the model specified. When a component is exogenous as well as endogenous, for example,  $\gamma_2$  in (2),  $\mathbf{V}$  and  $\mathbf{W}$  share a column of the same weights for the component. The proposed model deals with components or weighted composites of observed variables rather than factors. This is similar to PLS. It

is straightforward to capture both formative and reflective indicators in the model as seen in (3), which is also comparable to PLS. All PLS models are composed of two submodels such as the structural (inner) model between components and the measurement (outer) model between observed variables and components. In PLS, the submodels entail their own optimization criterion to be minimized separately for parameter estimation. On the other hand, the proposed model deals with such submodels in a unified algebraic formulation, so that it can construct a single optimization criterion for parameter estimation, allowing an overall measure of model fit.

## 2.2. Parameter Estimation

We estimate the unknown parameters  $\mathbf{V}$ ,  $\mathbf{W}$ , and  $\mathbf{A}$  in such a way that the sum of squares of the residuals,  $\mathbf{E} = \mathbf{ZV} - \mathbf{ZWA} = \mathbf{\Psi} - \mathbf{\Gamma A}$ , is as small as possible. This amounts to minimizing

$$\begin{aligned} f &= \text{SS}(\mathbf{ZV} - \mathbf{ZWA}) \\ &= \text{SS}(\mathbf{\Psi} - \mathbf{\Gamma A}), \end{aligned} \quad (4)$$

with respect to  $\mathbf{V}$ ,  $\mathbf{W}$ , and  $\mathbf{A}$ , where  $\text{SS}(\mathbf{X}) = \text{trace}(\mathbf{X}'\mathbf{X})$ . The components in  $\mathbf{\Psi}$  and/or  $\mathbf{\Gamma}$  are subject to normalization for identification purposes, for example,  $\gamma_1' \gamma_1 = 1$  in (2).

We cannot solve (4) in an analytic way since  $\mathbf{V}$ ,  $\mathbf{W}$ , and  $\mathbf{A}$  can comprise zero or any fixed elements. Instead, we develop an alternating least squares (ALS) algorithm (de Leeuw, Young, & Takane, 1976) to minimize (4). In general, ALS can be viewed as a special type of the FP algorithm where the fixed point is a stationary (accumulation) point of a function to be optimized. The proposed ALS algorithm consists of two steps: In the first step,  $\mathbf{A}$  is updated for fixed  $\mathbf{V}$  and  $\mathbf{W}$ . In the second step,  $\mathbf{V}$  and  $\mathbf{W}$  are updated for fixed  $\mathbf{A}$ .

To update  $\mathbf{A}$ , in the first step, (4) can be re-written as

$$\begin{aligned} f &= \text{SS}(\text{vec}(\mathbf{\Psi}) - \text{vec}(\mathbf{\Gamma A})) \\ &= \text{SS}(\text{vec}(\mathbf{\Psi}) - (\mathbf{I} \otimes \mathbf{\Gamma})\text{vec}(\mathbf{A})), \end{aligned} \quad (5)$$

where  $\text{vec}(\mathbf{X})$  denotes a supervector formed by stacking all columns of  $\mathbf{X}$  one below another, and  $\otimes$  denotes a Kronecker product. Let  $\mathbf{a}$  denote the vector formed by eliminating zero elements from  $\text{vec}(\mathbf{A})$ . Let  $\mathbf{\Omega}$  denote the matrix formed by eliminating the columns of  $\mathbf{I} \otimes \mathbf{\Gamma}$  corresponding to the zero elements in  $\text{vec}(\mathbf{A})$ . Then, the least squares estimate of  $\mathbf{a}$  for fixed  $\mathbf{V}$  and  $\mathbf{W}$  is obtained by

$$\hat{\mathbf{a}} = (\mathbf{\Omega}'\mathbf{\Omega})^{-1}\mathbf{\Omega}'\text{vec}(\mathbf{\Psi}). \quad (6)$$

The updated  $\mathbf{A}$  is reconstructed from  $\hat{\mathbf{a}}$ . It is assumed that  $\mathbf{\Omega}'\mathbf{\Omega}$  is nonsingular. If it is not, we may use any g-inverse in (6). It is convenient to use the Moore–Penrose inverse to obtain a unique solution.

The second step updates  $\mathbf{V}$  and  $\mathbf{W}$  for fixed  $\mathbf{A}$ . As seen in (2) and (3), some columns of parameters are often duplicated over  $\mathbf{V}$  and  $\mathbf{W}$ , while other columns are included in either  $\mathbf{V}$  or  $\mathbf{W}$ . Therefore, each column in  $\mathbf{V}$  and  $\mathbf{W}$  should be separately updated. To make the problem simple, suppose for now that we only estimate a single parameter,  $\mathbf{s}$ , which is shared by  $\mathbf{V}$  and  $\mathbf{W}$ . Let  $\mathbf{v}_p$  and  $\mathbf{w}_q$  denote the  $p$ th and  $q$ th columns in  $\mathbf{V}$  and  $\mathbf{W}$ , respectively ( $p = 1, \dots, P$ ;  $q = 1, \dots, Q$ ). Suppose that  $\mathbf{v}_p$  and  $\mathbf{w}_q$  are common for  $\mathbf{V}$  and  $\mathbf{W}$ , so that  $\mathbf{s} = \mathbf{v}_p = \mathbf{w}_q$ . Let  $\mathbf{\Lambda} = \mathbf{W A}$ . Let  $\mathbf{V}_{(-p)}$  denote  $\mathbf{V}$  whose  $p$ th column is the vector of zeros. Let  $\mathbf{V}_{(p)}^*$  denote  $\mathbf{V}$  whose columns are all zero vectors except the  $p$ th column. Let  $\mathbf{\Lambda}_{(-q)}$  denote a product matrix of  $\mathbf{W}$  whose  $q$ th column is the vector of zeros and  $\mathbf{A}$  whose  $q$ th row is the zero vector. Let  $\mathbf{\Lambda}_{(q)}^*$  denote a product matrix of  $\mathbf{W}$  whose columns are all zero vectors except the  $p$ th column and  $\mathbf{A}$  whose rows are all zero vectors except the  $q$ th row. Let  $\mathbf{e}'_p$  denote a row vector whose elements

are all zeros except the  $p$ th element being unity. Let  $\mathbf{a}'_q$  denote the  $q$ th row of  $\mathbf{A}$ . To update  $\mathbf{s}$ , then, (4) can be re-expressed as

$$\begin{aligned}
f &= \text{SS}(\mathbf{Z}[\mathbf{V} - \mathbf{\Lambda}]) \\
&= \text{SS}(\mathbf{Z}[(\mathbf{V}_{(-p)} + \mathbf{V}_{(p)}^*) - (\mathbf{\Lambda}_{(-q)} + \mathbf{\Lambda}_{(q)}^*)]) \\
&= \text{SS}(\mathbf{Z}[(\mathbf{V}_{(p)}^* - \mathbf{\Lambda}_{(q)}^*) - (\mathbf{\Lambda}_{(-q)} - \mathbf{V}_{(-p)})]) \\
&= \text{SS}(\mathbf{Z}[\mathbf{s}(\mathbf{e}'_p - \mathbf{a}'_q) - \mathbf{\Delta}]) \\
&= \text{SS}(\text{vec}(\mathbf{Z}\mathbf{s}\boldsymbol{\beta}') - \text{vec}(\mathbf{Z}\mathbf{\Delta})) \\
&= \text{SS}((\boldsymbol{\beta}' \otimes \mathbf{Z})\mathbf{s} - \text{vec}(\mathbf{Z}\mathbf{\Delta})), \tag{7}
\end{aligned}$$

where  $\boldsymbol{\beta}' = \mathbf{e}'_p - \mathbf{a}'_q$ , and  $\mathbf{\Delta} = \mathbf{\Lambda}_{(-q)} - \mathbf{V}_{(-p)}$ .

Let  $P$  and  $Q$  denote the numbers of columns consisting of unknown parameters in  $\mathbf{V}$  and  $\mathbf{W}$ , respectively. Let  $U$  denote the number of common columns in  $\mathbf{V}$  and  $\mathbf{W}$ . Let  $K = P + Q - U$ . To update all parameters in  $\mathbf{V}$  and  $\mathbf{W}$ , then, (7) can be generalized as

$$f = \sum_{k=1}^K \text{SS}((\boldsymbol{\beta}' \otimes \mathbf{Z})\mathbf{s}_k - \text{vec}(\mathbf{Z}\mathbf{\Delta})). \tag{8}$$

In (8),  $\boldsymbol{\beta}'$  and  $\mathbf{\Delta}$  are defined as follows:

$$\boldsymbol{\beta}' = \begin{cases} \mathbf{e}'_p - \mathbf{a}'_q & \text{if } \mathbf{s}_k \text{ is shared by } \mathbf{V} \text{ and } \mathbf{W} \\ \mathbf{e}'_p & \text{if } \mathbf{s}_k = \mathbf{v}_p \\ \mathbf{a}'_q & \text{if } \mathbf{s}_k = \mathbf{w}_q \end{cases}, \tag{9}$$

and

$$\mathbf{\Delta} = \begin{cases} \mathbf{\Lambda}_{(-p)} - \mathbf{V}_{(-q)} & \text{if } \mathbf{s}_k \text{ is shared by } \mathbf{V} \text{ and } \mathbf{W} \\ \mathbf{\Lambda} - \mathbf{V}_{(-p)} & \text{if } \mathbf{s}_k = \mathbf{v}_p \\ \mathbf{V} - \mathbf{\Lambda}_{(-q)} & \text{if } \mathbf{s}_k = \mathbf{w}_q \end{cases} \tag{10}$$

Let  $\boldsymbol{\eta}_k$  denote the vector formed by eliminating any fixed elements from  $\mathbf{s}_k$ . Let  $\boldsymbol{\Pi}$  denote the matrix formed by eliminating the columns of  $\boldsymbol{\beta}' \otimes \mathbf{Z}$  corresponding to the fixed elements in  $\mathbf{s}_k$ . Then, the least squares estimate of  $\boldsymbol{\eta}_k$  is obtained by

$$\hat{\boldsymbol{\eta}}_k = (\boldsymbol{\Pi}'\boldsymbol{\Pi})^{-1}\boldsymbol{\Pi}'\text{vec}(\mathbf{Z}\mathbf{\Delta}). \tag{11}$$

The updated  $\mathbf{s}_k$  is recovered from  $\hat{\boldsymbol{\eta}}_k$ . In (8), we see that updating  $\mathbf{s}_k$  is dependent on other parameter estimates in  $\mathbf{V}$  and  $\mathbf{W}$ . To assure convergence, therefore, we must immediately replace the previously estimated parameter column by the newly estimated (and normalized if necessary) one. When  $\mathbf{s}_k$  is included in both  $\mathbf{V}$  and  $\mathbf{W}$ , the updated  $\mathbf{s}_k$  should be substituted for the corresponding columns in  $\mathbf{V}$  and  $\mathbf{W}$ . If  $\boldsymbol{\Pi}'\boldsymbol{\Pi}$  is singular, again, we may use the Moore–Penrose inverse to obtain a unique solution in (11).

We alternate the two main steps until convergence is reached, that is, until the decrease in the function value falls below a certain threshold value, say  $10^{-4}$ . A few remarks concerning the proposed algorithm are in order. First of all, the ALS algorithm monotonically decreases the value of criterion (4), which is also bounded from below. The algorithm is therefore convergent. However, it does not guarantee that the convergence point is the global minimum. This so-called convergence to nonglobal minimum problem may be avoided in two ways (e.g., ten Berge, 1993). When we choose good (or rational) initial values, the function value is likely to start near the

global minimum, and it is more likely to obtain the global minimum. In GSCA, we may apply constrained components analysis (Takane, Kiers, & de Leeuw, 1995) to  $\mathbf{Z}$ , which can be regarded as a component-based alternative to confirmatory factor analysis (Kiers, Takane, & ten Berge, 1996). We may then employ the resultant component weights as rational starts for  $\mathbf{V}$  and  $\mathbf{W}$ . The initial values for  $\mathbf{A}$  are simply obtained by the least squares estimate, given  $\mathbf{V}$  and  $\mathbf{W}$ . The second possible remedy is to repeat the ALS procedure with many random initial starts. The obtained function values after convergence are compared, and the solution associated with the smallest one is chosen.

When  $N$  is large relative to  $J$ , the above algorithm may be made more efficient by the following procedure. Let  $\mathbf{Z} = \mathbf{QR}'$  be a portion of the QR decomposition of  $\mathbf{Z}$ , pertaining to the column space of  $\mathbf{Z}$ , where  $\mathbf{Q}$  is an  $N$  by  $J$  orthonormal matrix, so that  $\mathbf{Q}'\mathbf{Q} = \mathbf{I}$ , and  $\mathbf{R}'$  is a  $J$  by  $J$  upper-triangular matrix. Then, (4) can be rewritten as follows:

$$\begin{aligned} f &= \text{SS}(\mathbf{QR}'\mathbf{V} - \mathbf{QR}'\mathbf{A}) \\ &= \text{SS}(\mathbf{Q}(\mathbf{R}'\mathbf{V} - \mathbf{R}'\mathbf{A})) \\ &= \text{SS}(\mathbf{R}'\mathbf{V} - \mathbf{R}'\mathbf{A}), \end{aligned} \quad (12)$$

It is computationally more efficient to minimize (12) instead of (4) because the size of  $\mathbf{R}'$  is usually much smaller than  $\mathbf{Z}$ . Moreover, this procedure allows us to use covariance or correlation matrices instead of data matrices because  $\mathbf{Z}'\mathbf{Z} = \mathbf{RR}'$ , where  $\frac{1}{N}\mathbf{Z}'\mathbf{Z}$  is a covariance matrix if  $\mathbf{Z}$  is columnwise-centered and a correlation matrix if  $\mathbf{Z}$  is standardized.

In GSCA, the overall fit of a hypothesized model is measured by the total variance of all the endogenous variables explained by the specified model predictions. This is given by

$$\text{Fit} = 1 - \frac{\text{SS}(\mathbf{\Psi} - \mathbf{\Gamma}\mathbf{A})}{\text{SS}(\mathbf{\Psi})}. \quad (13)$$

This fit index ranges from 0 to 1. The larger the fit value, the more variance of the endogenous variables is explained by the model. It is a function of the sum of the squared residuals that summarizes the discrepancies between the model and the data. This kind of overall fit measure allows the evaluation of the adequacy of the whole model (Bollen, 1989, p. 256) and makes it possible to compare different models. Nonetheless, it is also crucial to examine the local goodness of fit of individual parameter estimates (Bollen, 1989, p. 281). For example, we may check for the loadings (equal to the correlations between observed variables and their component) and squared multiple correlations (equal to the squared loadings) for individual observed variables to evaluate the adequacy of components. We may also look at the standard errors or confidence intervals of parameter estimates to examine their reliability. Besides such statistical measures of model fit, nonstatistical considerations such as model interpretability often play a role in model evaluation, although they are usually more difficult to justify since they are largely subjective (Browne & Cudeck, 1993, p. 136).

We may use resampling methods such as the jackknife and the bootstrap methods to calculate standard errors of parameter estimates. In GSCA, the standard errors are estimated by the bootstrap method (Efron, 1982). The bootstrapped standard errors or confidence intervals can be used to assess the reliability of the parameter estimates.

We can test a variety of hypotheses on parameters by incorporating linear constraints into the model. The linear constraints may be specified by either the reparametrization or the null-space method (Böckenholt & Takane, 1994; Takane, Yanai, & Mayekawa, 1991). The former method specifies the space spanned by column vectors of a constraint matrix, while the latter specifies its ortho-complement space. In GSCA, all linear constraints are imposed by the reparametrization method. For example, let  $\mathbf{M}$  denote a matrix of linear constraints on  $\mathbf{a}$ . In the step of (6), we







TABLE 1.  
The congruence coefficients between parameters and estimates obtained from different sample sizes (1,000 replications per sample size).

	Mean	S.D.	Min	Max
$N = 10$	0.908	0.103	0.432	0.997
$N = 30$	0.971	0.034	0.541	0.997
$N = 50$	0.981	0.013	0.848	0.997
$N = 75$	0.984	0.009	0.907	0.996
$N = 100$	0.986	0.007	0.940	0.996
$N = 200$	0.988	0.004	0.968	0.995

That is, the correlations between the residuals associated with the same component were equal to 0.3, the correlations between the residuals across different components were equal to 0.1, and the correlations between  $\mathbf{d}$  and other residuals were equal to zero. These population covariances were also selected on the basis of estimates obtained from previous studies with the same model. Furthermore, we assumed that  $\mathbf{E} \sim N(\mathbf{0}, \Sigma_E)$ . This led to  $\mathbf{Z} \sim N(\mathbf{0}, (\Phi\Phi')^{-1}\Phi\Sigma_E\Phi'(\Phi\Phi')^{-1})$ , where  $\Phi = \mathbf{V} - \mathbf{WA}$ , since (1) can be re-expressed as  $\mathbf{Z}\Phi = \mathbf{E}$ . Let  $\Sigma_Z = (\Phi\Phi')^{-1}\Phi\Sigma_E\Phi'(\Phi\Phi')^{-1}$ , and let  $\mathbf{R}\mathbf{R}' = \Sigma_Z$ . Let  $\mathbf{Y}$  denote an  $N$  by  $T$  matrix whose columns were generated from  $N(0, 1)$ . Then  $\mathbf{Z}$  could be generated from  $\mathbf{Y}\mathbf{R}$ .

In the Monte-Carlo simulation, sample sizes of  $\mathbf{Z}$  varied as follows: 10, 30, 50, 75, 100, and 200 observations. For each sample size, 1,000 replications were obtained. We used the true parameter values as the initial values for the ALS estimation in each replication so as to speed up convergence and to increase the possibility of convergence to the global minimum. We calculated the congruence coefficient (Tucker, 1951) between parameters and estimates as a recovery measure, which is defined as follows: Let  $\boldsymbol{\theta}$  and  $\boldsymbol{\rho}$  denote the vectors of the true parameters and the estimates from a single replication, respectively. Then, the congruence coefficient is given by  $\boldsymbol{\theta}'\boldsymbol{\rho}/\sqrt{\boldsymbol{\theta}'\boldsymbol{\theta}\boldsymbol{\rho}'\boldsymbol{\rho}}$ .

Table 1 provides the descriptive statistics of the recovery measure obtained from different sample sizes. GSCA converged in all cases. As seen in the table, even when the sample size is 10, the mean congruence coefficient between parameters and estimates (.908) is greater than .90, which is a conventional rule of thumb criterion as an acceptable degree of similarity (Mulaik, 1972). Therefore, the simulation results suggest that GSCA performs acceptably well in small samples in terms of recovery of parameters. In this study, indeed, 50 observations are likely to recover parameters sufficiently well since even the minimum congruence coefficient is close to .90 and also the mean congruence coefficient seems to increase slowly beyond the sample size.

#### 4. Application: The Organizational Identification Data

The present example is part of the organizational identification data used in Bergami and Bagozzi (2000). It consists of a sample of 305 employees (male = 157 and female = 148) from the electronics division of a large conglomerate in South Korea. From the data, we used 21 items associated with four components, such as organization prestige, organizational identification, affective commitment (joy), and affective commitment (love). The 21 items are presented in the Appendix. According to Bergami and Bagozzi (2000), organization prestige represents the perception of a member of the organization that their significant others (relatives, friends, and so on) believe that the organization is well-approved. Organizational identification is "a form of social identification whereby a person comes to view him- or herself as a member of a particular organization" (Bergami & Bagozzi, 2000, p. 557). Affective commitment (joy) and affective commitment (love) indicate two distinct types of emotional attachment to the organization: the

former (joy) is “happiness arising from the organization” (Bergami & Bagozzi, 2000, p. 560), and the latter (love) is “emotional attraction or affection towards the organization” (Bergami & Bagozzi, 2000, p. 560). We assumed a path model with the components, in which organization prestige was defined as a linear combination of eight items (org\_pre1–org\_pre8), organizational identification was as a linear combination of the six items proposed by Mael (1988) (org\_ident1–org\_ident6), and affective commitment (joy) and affective commitment (love) as linear combinations of four items (ac\_joy1–ac\_joy4) and three items (ac\_love1–ac\_love3), respectively, among the seven items for affective commitment, developed by Allen and Meyer (1990). Based on the findings in Bergami and Bagozzi (2000), we further assumed that organization prestige had an effect on organizational identification, and organizational identification had influences on both affective commitment (joy) and affective commitment (love). The specified model is presented in Figure 4.

To make the figure more concise, we present only component loadings ( $c_1$  to  $c_{21}$ ) and path coefficients ( $b_1$ ,  $b_2$ , and  $b_3$ ) here. The component weights, denoted by  $w_1$  to  $w_{21}$ , were also specified in the same order as the component loadings.

We applied GSCA to fit the specified model simultaneously to male and female groups in order to examine the differences in parameter estimates between the groups. We also applied PLS-Graph 3.0 (Chin, 2001), to the same data for the comparison of performance with GSCA. PLS-Graph was applied to fit each of the groups separately since PLS cannot make a single formulation for simultaneous analysis of the two groups such as (22). In both GSCA and PLS-Graph, we calculated the bootstrapped standard errors of all parameter estimates based on 100 bootstrap samples.

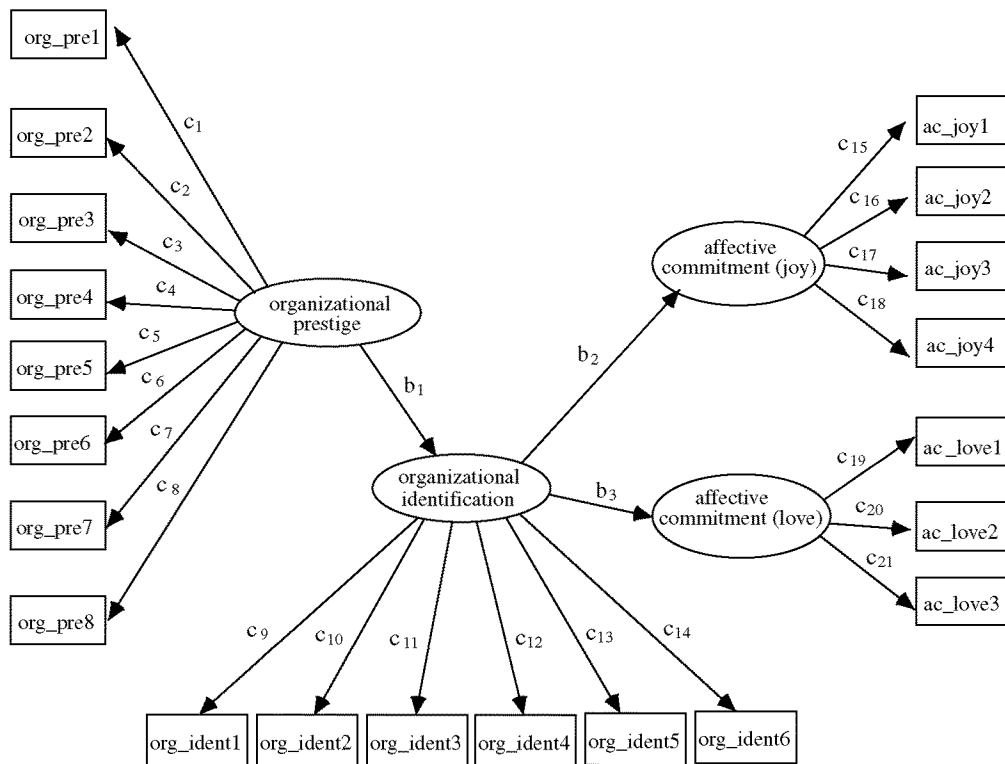


FIGURE 4.

The specified path model with components for the organizational identification data.

TABLE 2.

The component weight estimates and their standard errors in the parenthesis obtained from the unconstrained and constrained multi-group comparison analyses for the organizational identification data.

	GSCA				PLS-Graph	
	Unconstrained		Constrained		Unconstrained	
	Male	Female	Male	Female	Male	Female
$w_1$	.16 (.01)	.15 (.01)	.16 (.01)	.15 (.01)	.13 (.03)	.16 (.04)
$w_2$	.16 (.01)	.16 (.01)	.16 (.01)	.17 (.01)	.16 (.03)	.16 (.03)
$w_3$	.14 (.01)	.15 (.01)	.14 (.01)	.16 (.01)	.22 (.06)	.12 (.03)
$w_4$	.16 (.01)	.16 (.01)	.16 (.01)	.16 (.01)	.11 (.04)	.14 (.02)
$w_5$	.16 (.01)	.16 (.01)	.16 (.01)	.15 (.01)	.13 (.04)	.17 (.03)
$w_6$	.17 (.01)	.16 (.01)	.17 (.01)	.16 (.01)	.17 (.04)	.19 (.03)
$w_7$	.15 (.01)	.15 (.01)	.15 (.01)	.15 (.01)	.17 (.03)	.15 (.03)
$w_8$	.17 (.01)	.15 (.01)	.17 (.01)	.15 (.01)	.16 (.01)	.13 (.03)
$w_9$	.22 (.01)	.24 (.02)	.22 (.01)	.24 (.02)	.21 (.01)	.24 (.04)
$w_{10}$	.21 (.01)	.23 (.01)	.21 (.01)	.23 (.02)	.20 (.02)	.21 (.04)
$w_{11}$	.17 (.01)	.21 (.01)	.17 (.01)	.21 (.01)	.17 (.02)	.22 (.04)
$w_{12}$	.23 (.01)	.23 (.02)	.23 (.01)	.23 (.02)	.26 (.02)	.24 (.04)
$w_{13}$	.22 (.01)	.24 (.01)	.22 (.01)	.24 (.01)	.23 (.02)	.27 (.03)
$w_{14}$	.20 (.01)	.25 (.02)	.20 (.01)	.25 (.02)	.18 (.02)	.21 (.03)
$w_{15}$	.31 (.01)	.33 (.03)	.31 (.01)	.33 (.03)	.31 (.03)	.22 (.06)
$w_{16}$	.31 (.01)	.39 (.02)	.31 (.02)	.39 (.02)	.29 (.03)	.40 (.05)
$w_{17}$	.34 (.02)	.35 (.02)	.34 (.02)	.35 (.02)	.35 (.03)	.38 (.06)
$w_{18}$	.30 (.02)	.28 (.03)	.30 (.02)	.28 (.03)	.32 (.03)	.34 (.06)
$w_{19}$	.44 (.02)	.48 (.04)	.44 (.02)	.48 (.04)	.40 (.05)	.44 (.12)
$w_{20}$	.40 (.02)	.43 (.06)	.40 (.02)	.43 (.07)	.36 (.08)	.19 (.13)
$w_{21}$	.41 (.02)	.47 (.05)	.42 (.02)	.47 (.05)	.48 (.07)	.65 (.12)

GSCA provides that the overall goodness of fit of the specified model is equal to .60, indicating that about 60% of the total variance of all endogenous variables in both groups is accounted for by the specified model. The component weights and their bootstrapped standard errors estimated from GSCA and PLS-Graph are given in Table 2 (see the unconstrained solutions).

It is found that the standard errors of the estimates obtained from PLS-Graph seem to be bigger than those from GSCA. In PLS-Graph, the component weight estimate for  $ac\_love2$  in the female group ( $w_{20} = .19$ ) appears to be much smaller (and nonsignificant) compared to those for  $ac\_love1$  ( $w_{19} = .44$ ) and  $ac\_love3$  ( $w_{21} = .65$ ). This seems to be inconsistent with the fact that the correlations among the three observed variables are similar, that is,  $\text{corr}(ac\_love1, ac\_love2) = .27$ ,  $\text{corr}(ac\_love1, ac\_love3) = .32$ , and  $\text{corr}(ac\_love2, ac\_love3) = .26$ . On the other hand, the component weight estimates for the same variables look similar to one another and are all significant in GSCA. Despite these differences between the methods, overall, they provide quite similar component weight estimates for the two groups. In general, the weight estimates for each component are similar to one another and significant in both groups, indicating that all of them contribute equally well to determining their components.

The component loadings and their bootstrapped standard errors estimated from GSCA and PLS-Graph are given in Table 3 (see the unconstrained solutions). Again, both methods provide quite similar loading estimates overall, although the loading estimate for  $ac\_love2$  in the female group ( $c_{20} = .49$ ) looks smaller than those for  $ac\_love1$  ( $c_{19} = .75$ ) and  $ac\_love3$  ( $c_{21} = .85$ ) in PLS-Graph. In both male and female groups, the loading estimates of all components are high and significant. Also, the squared multiple correlations (corresponding to the squared load-

TABLE 3.

The component loading estimates and their standard errors in the parenthesis obtained from the unconstrained and constrained multi-group comparison analyses for the organizational identification data.

	GSCA				PLS-Graph	
	Unconstrained		Constrained		Unconstrained	
	Male	Female	Male	Female	Male	Female
$c_1$	.80 (.04)	.77 (.03)	.80 (.04)	.76 (.03)	.78 (.05)	.77 (.04)
$c_2$	.78 (.04)	.86 (.02)	.79 (.04)	.85 (.02)	.79 (.04)	.85 (.02)
$c_3$	.71 (.05)	.81 (.03)	.72 (.05)	.81 (.03)	.76 (.04)	.79 (.04)
$c_4$	.80 (.06)	.82 (.04)	.79 (.06)	.81 (.03)	.77 (.08)	.82 (.04)
$c_5$	.80 (.04)	.82 (.04)	.79 (.04)	.80 (.04)	.77 (.06)	.82 (.04)
$c_6$	.84 (.04)	.84 (.04)	.84 (.03)	.84 (.04)	.85 (.04)	.85 (.04)
$c_7$	.76 (.05)	.78 (.03)	.76 (.04)	.79 (.03)	.79 (.04)	.78 (.05)
$c_8$	.84 (.03)	.78 (.05)	.83 (.04)	.77 (.05)	.82 (.05)	.77 (.07)
$c_9$	.84 (.03)	.74 (.04)	.84 (.03)	.73 (.04)	.84 (.04)	.72 (.04)
$c_{10}$	.80 (.03)	.71 (.05)	.80 (.03)	.71 (.05)	.79 (.04)	.70 (.06)
$c_{11}$	.63 (.05)	.64 (.05)	.62 (.06)	.64 (.05)	.64 (.06)	.65 (.05)
$c_{12}$	.86 (.02)	.72 (.06)	.86 (.02)	.71 (.07)	.87 (.02)	.74 (.07)
$c_{13}$	.84 (.03)	.73 (.05)	.84 (.03)	.73 (.05)	.85 (.03)	.75 (.04)
$c_{14}$	.77 (.04)	.76 (.05)	.77 (.04)	.76 (.05)	.76 (.05)	.74 (.06)
$c_{15}$	.78 (.04)	.72 (.05)	.79 (.04)	.72 (.05)	.79 (.05)	.64 (.08)
$c_{16}$	.77 (.03)	.83 (.03)	.77 (.04)	.84 (.03)	.77 (.04)	.83 (.04)
$c_{17}$	.85 (.02)	.75 (.05)	.84 (.03)	.75 (.04)	.85 (.02)	.78 (.05)
$c_{18}$	.76 (.04)	.60 (.08)	.76 (.04)	.60 (.08)	.77 (.04)	.64 (.07)
$c_{19}$	.84 (.03)	.75 (.05)	.84 (.03)	.75 (.05)	.83 (.05)	.75 (.11)
$c_{20}$	.77 (.04)	.67 (.10)	.76 (.05)	.67 (.11)	.75 (.08)	.49 (.16)
$c_{21}$	.79 (.04)	.74 (.07)	.80 (.04)	.74 (.06)	.83 (.04)	.85 (.07)

TABLE 4.

The path coefficient estimates and their standard errors in the parenthesis obtained from the unconstrained and constrained multi-group comparison analyses for the organizational identification data.

	GSCA				PLS-Graph	
	Unconstrained		Constrained		Unconstrained	
	Male	Female	Male	Female	Male	Female
$b_1$	.37 (.09)	.35 (.07)	.36 (.06)	.36 (.06)	.40 (.08)	.35 (.05)
$b_2$	.70 (.04)	.46 (.05)	.69 (.05)	.46 (.05)	.71 (.04)	.48 (.06)
$b_3$	-.45(.09)	-.31(.07)	-.45(.08)	-.31(.08)	-.46(.08)	-.36(.06)

ings) of the observed variables all turn out to be significant. These indicate that the components seem to be well constructed to account for the large portion of the variances of the observed variables.

Table 4 shows the path coefficient estimates for both groups obtained from GSCA and PLS-Graph (also see the unconstrained solutions). Both methods provide quite similar path coefficient estimates for the male and female groups, leading to essentially the same interpretations. In both groups, organizational prestige has a significant and positive effect on organizational identification. It suggests that the more both male and female employees perceive that important others believe that their organization is well-regarded, the more they perceive themselves as a member of the organization. In addition, organizational identification has a positive and significant

influence on affective commitment (joy) whereas a negative and significant effect on affective commitment (love) in both groups (note that the items on affective commitment (love) are described in a negative way). It suggests that in both groups the employees with high levels of organizational identification show high levels of the two types of emotional attachment to the organization.

It is shown from GSCA that the degree of the impact of organization prestige on organizational identification is similar in male (.37) and female (.35) groups. We thus decided to further hypothesize that the effect of organization prestige on organizational identification was equivalent across the two groups. This assumption was incorporated by imposing across-group equality constraints. As mentioned in section 2.3, PLS is unable to accommodate such a constrained multi-group comparison. Thus, we only applied GSCA for the constrained simultaneous analysis of the groups.

The component weight and loading estimates of the observed variables obtained from the constrained case are also given in Tables 2 and 3, respectively (see the constrained solutions in GSCA). All of the estimates are virtually identical to those from the unconstrained analysis. The path coefficient estimates from the constrained case are presented in Table 4 (also see the constrained solutions in GSCA). Due to imposing the equality constraints, the effect of organization prestige on organizational identification turns out to be identical across the two groups (.36). The other path coefficient estimates from the constrained case are almost identical to those from the unconstrained case. The overall goodness of fit of the constrained model is .60, which is essentially the same fit as that obtained from the unconstrained analysis. Therefore, it may be safe to say that our hypothesis regarding the parameters across the groups is reasonable. The constrained analysis yields simpler interpretations of the solutions, reducing the number of parameters to be estimated.

## 5. Concluding Remarks

In this paper, we proposed an alternative method, GSCA, to PLS for path analysis with components. GSCA can handle the relationships among components and observed variables in a unified algebraic framework. It leads to a well-defined least squares criterion to estimate model parameters, thus allowing an overall model fit measure. A straightforward estimation algorithm is developed to optimize the overall fitting criterion. According to our experience with simulation and actual data, the algorithm seems to be efficient. It converges fast, and seems to be hardly afflicted by the nonglobal minimum problem.

It was observed that GSCA offered quite similar results to those obtained from PLS, while providing additional advantages such as an overall model fit measure and multi-group comparisons with across-group constraints. It suggests that GSCA may be a suitable alternative to PLS. However, further studies are necessary for more careful comparisons between the methods. In particular, it may be useful to conduct a simulation study on the performance of the methods, considering a variety of experimental conditions such as sample size, model complexity, and so forth.

Several component analysis methods with a global optimization procedure have been proposed to fit path models with components (e.g., Hwang & Takane, 2002; Takane et al., 1995). However, they are limited to a particular type of relationship only. For example, they cannot fit path models with endogenous components. McDonald (1996) proposed a fairly comprehensive alternative method to PLS. Nonetheless, his method was developed from the perspective of covariance structure analysis (Bock & Bargmann, 1966; Jöreskog, 1970), so that it typically assumes reflective indicators and suffers from the problem of factor score indeterminacy. On the other hand, GSCA can deal with both formative and reflective indicators in a straightforward way and also is free from the factor score indeterminacy problem. Furthermore, GSCA subsumes

existing standard multivariate techniques as special cases, for example, (multivariate) regression analysis, ANOVA, discriminant analysis, canonical correlation analysis, and (constrained) principal components analysis. This flexibility of GSCA broadens its capacity.

A number of topics may be considered to further enhance the capability of GSCA. For example, all observed variables are so far assumed to be continuous or numerical. However, GSCA may be extended to deal with categorical variables through certain data transformations. In particular, the optimal scaling approach (e.g., Young, 1981) seems to be attractive in the context of the method since it may be readily subjected to the ALS estimation procedure. It may also be important to handle missing observations, which frequently appear in large data sets. A least squares imputation approach (e.g., Gabriel & Zamir, 1979) seems to be applicable for GSCA since it can easily be made compatible with the ALS estimation procedure. The least squares imputation approach may be further combined with the bootstrap, yielding a nonparametric multiple imputation method that takes into account a possibility of the uncertainty related to estimation of missing observations (e.g., Efron, 1994). This nonparametric multiple imputation approach is more consistent with GSCA, which is basically a distribution-free technique, rather than certain parametric multiple imputation methods such as data augmentation (Schafer, 1997). In addition, it is necessary to study robust estimation since GSCA may not be robust against outliers as far as it is based on solving a simple (unweighted) least squares criterion, which amounts to minimizing the sum of the "squared" residuals. Griep, Walkeling, Vankeerberghen, and Massart (1995) reported that the iteratively reweighted least squares (IRLS) method (Beaton & Tukey, 1974) performed better at least for the low dimensional PLS than other robust estimation methods. Therefore, IRLS may be a good candidate to handle outliers in GSCA that usually assumes a single component from a set of observed variables. Future research is needed to study the feasibility of these extensions.

## Appendix

### *The 21 Items Obtained from the Organizational Identification Data in Bergami and Bagozzi (2000).*

For each of the following items, please indicate how much you agree or disagree. Use the following 5-point scale: 1: strongly disagree; 2: disagree; 3: neither disagree or agree; 4: agree; 5: strongly agree.

#### **Organization prestige**

org\_pre1: My relatives and people close or important to me believe that [Company X] is a well-known company.

org\_pre2: My relatives and people close or important to me believe that [Company X] is a highly respected company.

org\_pre3: My relatives and people close or important to me believe that [Company X] is an admired company.

org\_pre4: My relatives and people close or important to me believe that [Company X] is a prestigious company.

org\_pre5: People in general think that [Company X] is a well-known company.

org\_pre6: People in general think that [Company X] is a highly respected company.

org\_pre7: People in general think that [Company X] is an admired company.

org\_pre8: People in general think that [Company X] is a prestigious company.

**Organizational identification**

org\_ident1: When someone criticizes [Company X] it feels like a personal insult.

org\_ident2: I am very interested in what others think about [Company X].

org\_ident3: When I talk about [Company X], I usually say “we” rather than “they”.

org\_ident4: [Company X’s] successes are my successes.

org\_ident5: When someone praises [Company X] it feels like a personal compliment.

org\_ident6: If a story in the media criticized [Company X], I would feel embarrassed.

**Affective commitment (joy)**

ac\_joy1: I would be very happy to spend the rest of my career with [Company X].

ac\_joy2: I enjoy discussing [Company X] with people outside of it.

ac\_joy3: I really feel the problems of [Company X] are my own.

ac\_joy4: [Company X] has a great deal of personal meaning for me.

**Affective commitment (love)**

ac\_love1: I do not feel like part of a family at [Company X].

ac\_love2: I do not feel emotionally attached to [Company X].

ac\_love3: I do not feel a strong sense of belonging to [Company X].

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