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GLOBAL LEAST SQUARES PATH MODELING: A FULL-INFORMATION ALTERNATIVE TO PARTIAL LEAST SQUARES PATH MODELING

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Partial least squares path modeling has been widely used for component-based structural equation modeling, where constructs are represented by weighted composites or components of observed variables. This approach remains a limited-information method that carries out two separate stages sequentially to estimate parameters (component weights, loadings, and path coefficients), indicating that it has no single optimization criterion for estimating the parameters at once. In general, limited-information methods are known to provide less efficient parameter estimates than full-information ones. To address this enduring issue, we propose a full-information method for partial least squares path modeling, termed global least squares path modeling, where a single least squares criterion is consistently minimized via a simple iterative algorithm to estimate all the parameters simultaneously. We evaluate the relative performance of the proposed method through the analyses of simulated and real data. We also show that from algorithmic perspectives, the proposed method can be seen as a block-wise special case of another full-information method for component-based structural equation modeling—generalized structured component analysis.

Key words: partial least squares path modeling, full-information, single optimization criterion, alternating least squares, block-wise generalized structured component analysis, component-based structural equation modeling, regularized generalized canonical correlation analysis, Lohmöller's algorithm, Wold's algorithm.

1. Introduction

Partial least squares path modeling (PLSPM; Lohmöller, 1989; Wold, 1966, 1973, 1982) is a well-known statistical method for component-based structural equation modeling (SEM), where constructs are represented by weighted composites of observed variables (i.e., components). Component-based SEM is different from factor-based SEM, in which constructs are represented by common factors (e.g., Jöreskog & Wold 1982; Rigdon, 2012; Rigdon, Sarstedt, & Ringle, 2017; M. Tenenhaus, 2008). Covariance structure analysis (CSA; Jöreskog, 1970, 1978) has been a standard method for factor-based SEM, although other methods, such as consistent partial least squares (PLSc; Dijkstra, 2010; Dijkstra & Henseler, 2015) and generalized structured component analysis with measurement errors incorporated ($GSCA_M$; Hwang, Takane, & Jung, 2017), can also be used for estimating factor-based models. It has been recognized that the two SEM domains would need to conceptually differentiate from each other (e.g., Hair, Hult, Ringle, Sarstedt, & Thiele, 2017; Rigdon, 2012) and their statistical methods should be used for estimating models with their corresponding representations of constructs (i.e., CSA for factor-based models and PLSPM for component-based models), otherwise tending to yield biased solutions (e.g., Cho, Sarstedt, & Hwang, 2020; Hwang, Cho, Jung, Falk, Flake, Jin, & Lee, in press; Sarstedt, Hair, Ringle, Thiele, & Gudergan, 2016). The focus of this paper is on PLSPM for component-based SEM.

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In model specification, PLSPM consists of two sub-models—measurement and structural models. The measurement model relates each block of observed variables to a component, whereas the structural model relates components among themselves. In parameter estimation, PLSPM applies two estimation stages "sequentially." The first stage estimates component weights assigned to a block of observed variables at a time, producing all block-wise components. The second stage then carries out a series of linear regression analyses to estimate remaining parameters, such as path coefficients and loadings, in the sub-models based on the components from the first stage. Specifically, path coefficients are estimated by regressing each dependent component on its independent components, whereas loadings are by regressing each block of observed variables on its component. The second stage hence involves non-iterative estimation.

On the other hand, the first stage requires an iterative algorithm to estimate component weights. Lohmöller's (1989) algorithm has been the de facto standard and implemented into various PLSPM software programs, including LVPLS (Lohmöller, 1984), PLS Graph (Chin, 2001), SmartPLS (Ringle, Wende, & Will, 2005), and XLSTAT (Addinsoft, 2009), although other algorithm (e.g., Wold, 1985) is also available (also see Hanafi, 2007; Tenenhaus & Tenenhaus, 2011). As will be discussed in detail in Sect. 2, Lohmöller's algorithm repeats two steps, called internal and external estimation. The internal estimation step updates the so-called inner estimate for each component in several ways, termed schemes, whereas the external estimation step updates component weights for each block of observed variables in two different ways, named Modes A and B. This algorithm seems to converge quickly in practice, generally providing unbiased estimates of component-based structural equation models (e.g., Cho & Choi, 2020; Dijkstra, 2017; Sarstedt et al., 2016). Nevertheless, it has been criticized mainly because there is no explicit optimization criterion that his algorithm aims to minimize or maximize by repeating the two steps under both modes (e.g., Coolen & de Leeuw, 1987; Jöreskog & Wold, 1982), although some researchers have shown that Lohmöller's algorithm appears to optimize a correlation maximization criterion at least under Mode B yet in a less optimal fashion, i.e., with no guarantee of monotonic convergence (e.g., Hanafi, 2007). The lack of a clear optimization criterion makes it difficult to evaluate the algorithm (McDonald, 1996). For example, it is unclear how the different schemes were theoretically derived and in what sense they are optimal for estimating the inner estimates (Hwang, Takane, & Tenenhaus, 2015). More importantly, it is difficult to evaluate in what sense the component weights estimated via Lohmöller's algorithm can be optimal under both modes (e.g., Rönkkö, McIntosh, Antonakis, & Edwards, 2016).

To address this issue with Lohmöller's algorithm, several researchers have proposed a single optimization criterion for estimating component weights in the first stage. For example, Hanafi (2007) proposed a correlation maximization criterion for Mode B, which is an extension of Wold's (1985) algorithm to additional schemes. Tenenhaus and Tenenhaus (2011) developed a multi-block component analysis method, called regularized generalized canonical correlation analysis (RGCCA), which involves a single covariance/correlation maximization criterion for estimating component weights in a variety of multi-block component analyses (Tenenhaus, Tenenhaus, & Groenen, 2017). Although RGCCA is not designed for component-based SEM, it can produce quite similar component weight estimates to those from Lohmöller's algorithm under Mode B. This indicates that the RGCCA criterion can serve as a single, well-defined optimization criterion for Mode B (Tenenhaus et al., 2017). Moreover, Hwang et al. (2015) proposed a single least squares criterion for estimating PLSPM's component weights. Their method was found to perform similarly to Lohmöller's algorithm, resulting in comparable or identical solutions under both Mode A and Mode B.

These methods are significant technical developments in PLSPM or multi-block analysis, addressing the main issue of Lohmöller's algorithm partly or entirely. Nevertheless, they focus solely on refining or replacing Lohmöller's algorithm that is used only for estimating component weights in the first estimation stage of PLSPM. Thus, PLSPM still remains a two-stage, sequential approach, indicating that it is not equipped with a single optimization criterion to estimate all parameters (i.e., component weights, loadings, and path coefficients) at the same time. PLSPM is hence considered a limited-information method that utilizes a subset of the entire system of equations at a time for estimating parameters (Tenenhaus, 2008). In general, limited-information methods tend to provide less efficient parameter estimates than full-information methods that estimate all parameters simultaneously, using information available in entire equations (e.g., Fomby, Johnson, & Hill, 2012, Chapter 22).

In this paper, we propose a full-information alternative to PLSPM, where a single least squares criterion is consistently minimized to estimate all the parameters simultaneously. We develop an alternating least squares (ALS) algorithm to minimize the single optimization criterion. We call the proposed method *global least squares path modeling* (GLSPM).[1] We will also show that from algorithmic perspectives, GLSPM can be viewed as a block-wise special case of another full-information method for component-based SEM—generalized structured component analysis (GSCA; Hwang & Takane, 2004, 2014).

The paper is organized as follows. We begin by describing existing PLSPM methods, which apply Lohmöller's algorithm and Hwang et al.'s (2015) procedure in the first stage, to facilitate the derivation of GLSPM. We concentrate on these two limited-information methods for PLSPM because as stated earlier, they update component weights in the same fashion under both Modes A and B and perform quite similarly in practice. Also, we briefly discuss RGCCA and compare it to Hanafi's (2007) and Hwang et al.'s procedures. As noted above, RGCCA and Hanafi's procedure can be used in lieu of Lohmöller's algorithm under Mode B. Next, we provide the technical underpinnings of GLSPM, including the proposed single optimization criterion and ALS algorithm. We subsequently conduct a simulation study to compare the performance of GLSPM and various extant PLSPM methods under different experimental conditions. We also apply the proposed method to real data to further examine its empirical performance. Lastly, we summarize the previous sections and discuss the implications of GLSPM.

2. Existing Methods for Partial Least Squares Path Modeling

Let $\mathbf{Z}_j$ denote an N by P_j matrix of the jth block of observed variables ($j = 1, 2, \ldots\ldots\ldots, J$). Let $\boldsymbol{\gamma}_j = \mathbf{Z}_j\mathbf{w}_j$ denote an N by 1 vector of the jth component, where $\mathbf{w}_j$ is a P_j by 1 vector of block-wise component weights assigned to $\mathbf{Z}_j$. The measurement model for PLSPM can be generally expressed as follows.

$$\mathbf{Z}_j = \boldsymbol{\gamma}_j\mathbf{c}_j{}' + \boldsymbol{\Pi}_j, \tag{1}$$

where $\mathbf{c}_j$ is a P_j by 1 vector of loadings and $\boldsymbol{\Pi}_j$ is an N by P_j matrix of residuals for $\mathbf{Z}_j$. This model is also called the reflective or outwards model. Although PLSPM considers other types of measurement model, such as the formative or inwards directed model and the multiple indicator/multiple causes model, it is unclear how and whether PLSPM can indeed estimate parameters for these measurement models. Thus, we focus on the measurement model (1).

Let $\boldsymbol{\Phi}_j$ denote an N by S_j matrix of independent components that have direct effects on $\boldsymbol{\gamma}_j$. The structural model for PLSPM can be generally expressed as

$$\boldsymbol{\gamma}_j = \boldsymbol{\Phi}_j\mathbf{b}_j + \boldsymbol{\delta}_j, \tag{2}$$

[1] The authors thank the Associate Editor for suggesting this term.

where $\mathbf{b}_j$ is an S_j by 1 vector of path coefficients relating $\mathbf{\Phi}_j$ to $\boldsymbol{\gamma}_j$ and $\boldsymbol{\delta}_j$ is an N by 1 vector of residuals for $\boldsymbol{\gamma}_j$.

PLSPM involves three sets of parameters—component weights ($\mathbf{w}_j$), loadings ($\mathbf{c}_j$), and path coefficients ($\mathbf{b}_j$). It applies two stages consecutively to estimate these parameters. As described below, the first stage estimates component weights in an iterative manner, thereby producing components, whereas the second estimates loadings and path coefficients non-iteratively by means of a series of linear regression analyses.

2.1. Estimation Stage 1

As we noted earlier, Lohmöller's algorithm has been the best-known procedure for the first stage. We thus begin by describing his algorithm. Conventionally, both observed variables and components are assumed to be standardized to have zero means and unit variances (e.g., $\boldsymbol{\gamma}_j{}'\boldsymbol{\gamma}_j = N$). However, we here assume that they are normalized to have their length equal to one (e.g., $\boldsymbol{\gamma}_j{}'\boldsymbol{\gamma}_j = 1$), which renders the exposition of equations more concise, while producing the same parameter estimates. The individual scores of standardized components can be obtained by multiplying their normalized scores by $\sqrt{N}$.

The Lohmöller algorithm starts with assigning arbitrary initial values to the block-wise weight vector $\mathbf{w}_j$ and producing the normalized component $\boldsymbol{\gamma}_j = \mathbf{Z}_j\mathbf{w}_j^*$, where $\mathbf{w}_j^* = \mathbf{w}_j/\sqrt{\mathbf{w}_j{}'\mathbf{Z}_j{}'\mathbf{Z}_j\mathbf{w}_j}$. It then repeats the following two steps.

Step 1 (internal estimation) We update the inner estimate for $\boldsymbol{\gamma}_j$. The inner estimate, denoted by $\mathbf{f}_j$, is a weighed sum of the components linked to $\boldsymbol{\gamma}_j$, which include those affecting $\boldsymbol{\gamma}_j$ and those being affected by $\boldsymbol{\gamma}_j$ in a given structural model. It is generally obtained as

$$\mathbf{f}_j = \sum_{q=1}^{Q_j} e_{jq}\boldsymbol{\gamma}_q = \mathbf{\Gamma}_j\mathbf{e}_j, \tag{3}$$

where $\boldsymbol{\gamma}_q$ is a component linked to $\boldsymbol{\gamma}_j$, e_{jq} is called the inner weight assigned to $\boldsymbol{\gamma}_q$, $\mathbf{\Gamma}_j$ is an N by Q_j matrix consisting of all Q_j components ($\boldsymbol{\gamma}_q$'s) connected to $\boldsymbol{\gamma}_j$, and $\mathbf{e}_j$ is a Q_j by 1 vector of inner weights (e_{jq}'s) assigned to $\mathbf{\Gamma}_j$. To illustrate (3), we consider a prototype structural model that includes four components, displayed in Fig. 1. For this prototype, the inner estimate of each component is given as

$$\begin{aligned}
\mathbf{f}_1 &= e_{13}\boldsymbol{\gamma}_3 = \mathbf{\Gamma}_1\mathbf{e}_1\\
\mathbf{f}_2 &= e_{23}\boldsymbol{\gamma}_3 = \mathbf{\Gamma}_2\mathbf{e}_2\\
\mathbf{f}_3 &= e_{31}\boldsymbol{\gamma}_1 + e_{32}\boldsymbol{\gamma}_2 + e_{34}\boldsymbol{\gamma}_4 = \mathbf{\Gamma}_3\mathbf{e}_3\\
\mathbf{f}_4 &= e_{43}\boldsymbol{\gamma}_3 = \mathbf{\Gamma}_4\mathbf{e}_4,
\end{aligned} \tag{4}$$

where $\mathbf{\Gamma}_1 = \mathbf{\Gamma}_2 = \mathbf{\Gamma}_4 = \boldsymbol{\gamma}_3, \mathbf{\Gamma}_3 = [\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\gamma}_4], \mathbf{e}_1 = e_{13}, \mathbf{e}_2 = e_{23}, \mathbf{e}_3 = [e_{31}; e_{32}; e_{34}]$, and $\mathbf{e}_4 = e_{43}$. Thus, the inner estimate can be conceptually seen as a summary measure that represents homogeneity of all components adjacent to each component prescribed in a structural model. As shown in (3), updating $\mathbf{f}_j$ is equivalent to updating its inner weights (e_{jq}'s) given components. There are three main ways, so-called schemes, for updating the inner weights: centroid (Wold, 1982), factorial (Lohmöller, 1989), and path weighting (Lohmöller, 1989). The centroid scheme uses the sign of the correlation between $\boldsymbol{\gamma}_q$ and $\boldsymbol{\gamma}_j$ as each inner weight, whereas the factorial scheme the correlation between $\boldsymbol{\gamma}_q$ and $\boldsymbol{\gamma}_j$. On the other hand, the path weighting scheme considers the direction of the relationships between components. Specifically, the inner weights are the

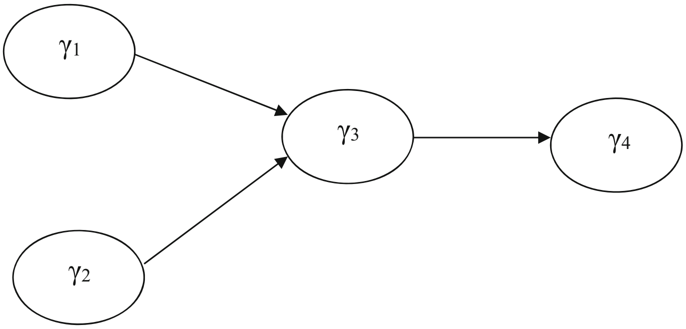

FIGURE 1.
A prototype structural model that involves four components. No residual terms are displayed

regression coefficients of $\boldsymbol{\gamma}_j$ on $\boldsymbol{\gamma}_q$'s in the case that $\boldsymbol{\gamma}_j$ is a dependent variable, whereas they are the correlations between $\boldsymbol{\gamma}_q$'s and $\boldsymbol{\gamma}_j$ in the case that $\boldsymbol{\gamma}_j$ is an independent variable.

As an example, we consider the following correlation matrix of the four components in the prototype model. If the centroid scheme is adopted for all the components, e_{13} and e_{31} are the

	$\boldsymbol{\gamma}_1$	$\boldsymbol{\gamma}_2$	$\boldsymbol{\gamma}_3$	$\boldsymbol{\gamma}_4$
$\boldsymbol{\gamma}_1$	1.0	−.2	−.5	−.2
$\boldsymbol{\gamma}_2$		1.0	.6	.3
$\boldsymbol{\gamma}_3$			1.0	.4
$\boldsymbol{\gamma}_4$				1.0

sign of the correlation between $\boldsymbol{\gamma}_1$ and $\boldsymbol{\gamma}_3$ (i.e., $e_{13} = e_{31} = -1$), e_{23} and e_{32} are the sign of the correlation between $\boldsymbol{\gamma}_2$ and $\boldsymbol{\gamma}_3$ (i.e., $e_{23} = e_{32} = 1$), and e_{34} and e_{43} are the sign of the correlation between $\boldsymbol{\gamma}_3$ and $\boldsymbol{\gamma}_4$ (i.e., $e_{34} = e_{43} = 1$). If the factorial scheme is used for the components, e_{13} and e_{31} are equivalent to the correlation between $\boldsymbol{\gamma}_1$ and $\boldsymbol{\gamma}_3$ (i.e., $e_{13} = e_{31} = -.5$), e_{23} and e_{32} are the correlation between $\boldsymbol{\gamma}_2$ and $\boldsymbol{\gamma}_3$ (i.e., $e_{23} = e_{32} = .6$), and e_{34} and e_{43} are the correlation between $\boldsymbol{\gamma}_3$ and $\boldsymbol{\gamma}_4$ (i.e., $e_{34} = e_{43} = .4$). If the path weighting scheme is chosen for the four components, e_{31} and e_{32} are the regression coefficients of $\boldsymbol{\gamma}_3$ on $\boldsymbol{\gamma}_1$ and $\boldsymbol{\gamma}_2$ (i.e., $e_{31} = -.40$ and $e_{32} = .52$) because $\boldsymbol{\gamma}_1$ and $\boldsymbol{\gamma}_2$ are independent variables for $\boldsymbol{\gamma}_3$, whereas e_{34} is the correlation between $\boldsymbol{\gamma}_3$ and $\boldsymbol{\gamma}_4$ (i.e., $e_{34} = .4$) because $\boldsymbol{\gamma}_3$ is an independent variable for $\boldsymbol{\gamma}_4$. The other inner estimates are the correlations between two connected components (i.e., $e_{13} = -.5$, $e_{23} = .6$, and $e_{43} = .4$) because the regression coefficient of one component on the other is equivalent to the correlation between them. The path weighting scheme has been recommended over the other schemes (Chin, 1998; Vinzi, Trinchera, & Amato, 2010). In practice, however, the choice of the three schemes seems to make little differences in the results (Lohmöller, 1989; Noonan & Wold, 1982). Although it is not commonly used, there is another scheme, called the Horst scheme (Krämer, 2007), where all inner weights per component are fixed to one. This scheme can be used when all components are positively correlated (Tenenhaus & Tenenhaus, 2011).

Step 2 (external estimation) We update $\mathbf{w}_j$. There are two ways of updating $\mathbf{w}_j$—Mode A and Mode B. Under Mode A, $\mathbf{w}_j$ is updated as follows.

$$\hat{\mathbf{w}}_j = \mathbf{Z}_j{}'\mathbf{f}_j. \tag{5}$$

Under Mode B, $\mathbf{w}_j$ is updated as follows.

$$\hat{\mathbf{w}}_j = (\mathbf{Z}_j{}'\mathbf{Z}_j)^{-1}\mathbf{Z}_j{}'\mathbf{f}_j. \tag{6}$$

Subsequently, the block-wise weight vector $\mathbf{w}_j$ is updated by $\hat{\mathbf{w}}_j^* = \hat{\mathbf{w}}_j / \sqrt{\hat{\mathbf{w}}_j' \mathbf{Z}_j{}'\mathbf{Z}_j \hat{\mathbf{w}}_j}$, and $\boldsymbol{\gamma}_j$ is updated by $\boldsymbol{\gamma}_j = \mathbf{Z}_j \hat{\mathbf{w}}_j^*$. The component weight estimates obtained under Mode A are called correlation weights, whereas those under Mode B called regression weights (Rigdon, 2012). Choices on the two modes are arbitrary and warrant further research (Dijkstra, 2017).

The two steps are repeated until no substantial differences occur between the previous and current weight estimates for all J blocks of observed variables. In this algorithm, it is not known what criterion is being optimized by iterating the two steps under both Mode A and Mode B. To address this issue, Hwang et al. (2015) proposed a single least squares criterion to estimate component weights under both modes. Let $\mathbf{e} = [\mathbf{e}_1; \ldots; \mathbf{e}_J]$ denote a Q by 1 vector consisting of all block-wise inner weights, where $Q = \sum_{j=1}^{J} Q_j$. Let $\mathbf{w} = [\mathbf{w}_1; \ldots; \mathbf{w}_J]$ denote a P by 1 vector of all block-wise component weights, where $P = \sum_{j=1}^{J} P_j$. Hwang et al.'s procedure aims to minimize the following criterion for the first stage.

$$\varphi(\mathbf{e}, \mathbf{w}) = \sum_{j=1}^{J} \alpha_j \mathrm{SS}(\mathbf{Z}_j - \mathbf{f}_j \mathbf{w}_j{}') + \sum_{j=1}^{J} (1 - \alpha_j) \mathrm{SS}(\mathbf{f}_j - \boldsymbol{\gamma}_j), \tag{7}$$

subject to $\boldsymbol{\gamma}_j'\boldsymbol{\gamma}_j = 1$, where $\mathrm{SS}(\mathbf{X}) = \mathrm{trace}(\mathbf{X}'\mathbf{X})$, and α_j denotes a binary value indicating which mode is used for each block of observed variables, i.e., $\alpha_j = 1$ for Mode A, and $\alpha_j = 0$ for Mode B. Given the inner estimate $\mathbf{f}_j$, the first term of (7) seems similar to a block-wise join loss function for principal component analysis (Gifi, 1990, p. 152), whereas the second term seems similar to a block-wise meet loss function for generalized canonical correlation analysis (Gifi, 1990, p. 167).

An ALS algorithm was developed to minimize (7). The ALS algorithm repeats the same estimation steps—internal and external. Unlike the Lohmöller algorithm, however, the two steps update both inner weights and component weights by minimizing (7) consistently. The ALS algorithm has been proved to converge (de Leeuw, Young, & Takane, 1976). It estimates both inner estimates and component weights optimally in the least squares sense (Hwang et al., 2015). Thus, Hwang et al. (2015) called their way of estimating the inner estimates the least squares scheme.

RGCCA aims to maximize a single optimization criterion that is the sum of some functional form of the covariance or correlation between two components over J blocks, with the imposition of normalization constraints on either component weights or components (Tenenhaus et al., 2017). Specifically, it seeks to maximize the following criterion.

$$\rho(\mathbf{w}) = \sum_{j,k=1, j \neq k}^{J} \lambda_{jk} \mathrm{g}\left[\mathrm{c}(\mathbf{Z}_j \mathbf{w}_j, \mathbf{Z}_k \mathbf{w}_k)\right], \tag{8}$$

subject to $\tau_j(\mathbf{w}_j' \mathbf{w}_j) + (1 - \tau_j)(\boldsymbol{\gamma}_j' \boldsymbol{\gamma}_j) = 1$, where τ_j is a so-called shrinkage constant taking a value in the interval [0,1], $\mathrm{c}(\mathbf{Z}_j \mathbf{w}_j, \mathbf{Z}_k \mathbf{w}_k)$ is the covariance or correlation between the jth and kth components ($j \neq k$), $\mathrm{g}(\cdot)$ denotes a function of the covariance or correlation (i.e., absolute, squared, and identity), and λ_{jk} is a binary value indicating whether the jth component is connected with the kth component in a given structural model, i.e., $\lambda_{jk} = 1$ if connected and otherwise $\lambda_{jk} = 0$. Tenenhaus & Tenenhaus (2011) showed that maximizing (8) is equivalent to maximizing the sum of a function of the covariance or correlation between the jth component and its inner estimate $\mathbf{f}_j$ over J blocks, and choosing the absolute, squared, and identity functions corresponds to adopting the centroid, factorial, and Horst schemes, respectively, for obtaining $\mathbf{f}_j$. They developed a monotonically convergent Gauss–Seidel-type algorithm, which carries out in a manner similar to Wold's (1985) algorithm or more generally Hanafi's (2007) procedure.

When RGCCA is applied with the imposition of the constraints $\boldsymbol{\gamma}_j'\boldsymbol{\gamma}_j = 1$ (i.e., $\tau_j = 0$), it is equivalent to estimating component weights under Mode B. Hanafi's (2007) procedure also aims to maximize (8) under the same constraints yet considers the centroid and factorial schemes only. Thus, RGCCA can include his procedure (and Wold's algorithm) as a special case, providing the same solutions under Mode B. It is also noteworthy that given the inner estimate $\mathbf{f}_j$, maximizing the RGCCA criterion with respect to $\mathbf{w}_j$, subject to $\boldsymbol{\gamma}_j'\boldsymbol{\gamma}_j = 1$, is equivalent to minimizing the second term of (7) only (Tenenhaus & Tenenhaus, 2011). A main difference is that RGCCA updates the inner estimates based on a predetermined scheme (e.g., centroid, factorial, etc.), whereas Hwang et al.'s (2015) procedure updates the inner estimates by consistently minimizing (7) without the need of predetermining a scheme.

On the other hand, when $\tau_j = 1$, RGCCA does not estimate component weights in the same way as in Lohmöller's algorithm under Mode A. This is mainly because RGCCA imposes different normalization constraints. As shown in (8), when $\tau_j = 1$, RGCCA estimates component weights with normalization constraints imposed on component weights (i.e., $\mathbf{w}_j'\mathbf{w}_j = 1$) rather than on components, thereby being unable to produce normalized components. Instead, RGCCA's way of estimating component weights subject to $\mathbf{w}_j'\mathbf{w}_j = 1$ is called New Mode A. When $0 < \tau_j < 1$, RGCCA provides regularized estimates of component weights. Although this way of estimating component weights, termed Mode Ridge, considers a continuum between Mode B and New Mode A, it is unclear how such a regularized case is directly related to PLSPM.

Thus, RGCCA can be chosen as a theoretically better alternative to Lohmöller's algorithm under Mode B (Tenenhaus & Tenenhaus, 2011). When Mode A is chosen for components, in theory, it might still be more appropriate to employ the conventional algorithm over RGCCA (Tenenhaus et al., 2017). Conversely, Hwang et al.'s (2015) procedure can provide normalized components under both Mode A and Mode B because as shown in (7), it always estimates component weights subject to $\boldsymbol{\gamma}_j'\boldsymbol{\gamma}_j = 1$, as in Lohmöller's algorithm.

2.2. *Estimation Stage 2*

Once all components are estimated from the first stage, the second stage estimates loadings and path coefficients by means of ordinary least squares regression. Specifically, the least squares estimates of $\mathbf{c}_j$ and $\mathbf{b}_j$ can be obtained as follows.

$$\hat{\mathbf{c}}_j = \mathbf{Z}_j{}'\boldsymbol{\gamma}_j \tag{9}$$

$$\hat{\mathbf{b}}_j = (\boldsymbol{\Phi}_j{}'\boldsymbol{\Phi}_j)^{-1}\boldsymbol{\Phi}_j{}'\boldsymbol{\gamma}_j. \tag{10}$$

3. Global Least Squares Path Modeling

We now propose a full-information method for PLSPM, termed GLSPM, which estimates all parameters (component weights, loadings, and path coefficients) simultaneously. We use the same notations for the parameters, components, and observed variables as those in the previous section. We also assume that all observed variables and components are normalized.

Let $\mathbf{c} = [\mathbf{c}_1; \ldots; \mathbf{c}_J]$ denote a P by 1 vector of all block-wise loadings. Let $\mathbf{b} = [\mathbf{b}_1; \ldots; \mathbf{b}_J]$ denote an S by 1 vector of all block-wise path coefficients, where $S = \sum_{j=1}^{J} S_j$. The objective of the proposed method is to combine the two estimation stages of PLSPM into a single framework. This can be achieved by minimizing the following least squares criterion.

$$\begin{aligned}\phi(\mathbf{e}, \mathbf{w}, \mathbf{c}, \mathbf{b}) &= \sum_{j=1}^{J} \left(\alpha_j \mathrm{SS}(\mathbf{Z}_j - \mathbf{f}_j \mathbf{w}_j{}') + (1-\alpha_j)\mathrm{SS}(\mathbf{f}_j - \boldsymbol{\gamma}_j) + \mathrm{SS}(\mathbf{Z}_j - \boldsymbol{\gamma}_j \mathbf{c}_j{}')\right. \\ &\quad \left. + \mathrm{SS}(\boldsymbol{\gamma}_j - \boldsymbol{\Phi}_j \mathbf{b}_j)\right), \qquad (11)\end{aligned}$$

subject to $\boldsymbol{\gamma}_j'\boldsymbol{\gamma}_j = 1$. This criterion is a mixture of Hwang et al.'s (2015) least squares criterion for the first stage and block-wise least squares criteria for estimating loadings and path coefficients in the second stage. It can be used for estimating component weights for each block of observed variables under either Mode A or Mode B by setting the corresponding α_j to one or zero, respectively.

We develop an ALS algorithm to minimize (11). The ALS algorithm begins by assigning arbitrary initial values (e.g., uniformly distributed random numbers in the interval [0,1]) to the block-wise weight vector $\mathbf{w}_j$ and obtaining the normalized component $\boldsymbol{\gamma}_j = \mathbf{Z}_j \mathbf{w}_j^*$, where $\mathbf{w}_j^* = \mathbf{w}_j / \sqrt{\mathbf{w}_j{}'\mathbf{Z}_j{}'\mathbf{Z}_j \mathbf{w}_j}$. Next, it alternates the following four steps, in each of which a set of parameters is updated with the other sets fixed.

Step 1 We update $\mathbf{e}_j$ with the other parameters fixed. This step is equivalent to minimizing

$$\phi_j^1(\mathbf{e}_j) = \alpha_j \mathrm{SS}(\mathbf{Z}_j - \boldsymbol{\Gamma}_j \mathbf{e}_j \mathbf{w}_j{}') + (1-\alpha_j)\mathrm{SS}(\boldsymbol{\Gamma}_j \mathbf{e}_j - \boldsymbol{\gamma}_j). \qquad (12)$$

By solving $\frac{1}{2}\frac{\partial \phi_j^1}{\partial \mathbf{e}_j} = \mathbf{0}$, the least squares estimate of $\mathbf{e}_j$ is obtained as

$$\hat{\mathbf{e}}_j = \left(\alpha_j \mathbf{w}_j{}'\mathbf{w}_j \boldsymbol{\Gamma}_j{}'\boldsymbol{\Gamma}_j + (1-\alpha_j)\boldsymbol{\Gamma}_j{}'\boldsymbol{\Gamma}_j\right)^{-1} \boldsymbol{\Gamma}_j{}'\boldsymbol{\gamma}_j. \qquad (13)$$

Then, $\mathbf{f}_j$ is updated by $\mathbf{f}_j = \boldsymbol{\Gamma}_j \hat{\mathbf{e}}_j$.

Step 2 We update $\mathbf{w}_j$ with the other parameters fixed. This is equivalent to minimizing

$$\begin{aligned}\phi_j^2(\mathbf{w}_j) &= \alpha_j \mathrm{SS}(\mathbf{Z}_j - \mathbf{f}_j \mathbf{w}_j{}') + (1-\alpha_j)\mathrm{SS}(\mathbf{f}_j - \boldsymbol{\gamma}_j) + \mathrm{SS}(\mathbf{Z}_j - \boldsymbol{\gamma}_j \mathbf{c}_j{}') + \mathrm{SS}(\boldsymbol{\gamma}_j - \boldsymbol{\Phi}_j \mathbf{b}_j) \\ &= \alpha_j \mathrm{SS}(\mathbf{Z}_j - \mathbf{f}_j \mathbf{w}_j{}') + (1-\alpha_j)\mathrm{SS}(\mathbf{f}_j - \mathbf{Z}_j \mathbf{w}_j) + \mathrm{SS}(\mathbf{Z}_j - \mathbf{Z}_j \mathbf{w}_j \mathbf{c}_j{}') + \mathrm{SS}(\mathbf{Z}_j \mathbf{w}_j - \boldsymbol{\Phi}_j \mathbf{b}_j) \\ &= \alpha_j \mathbf{w}_j{}'\mathbf{f}_j{}'\mathbf{f}_j \mathbf{w}_j + (1-\alpha_j)\mathbf{w}_j{}'\mathbf{Z}_j{}'\mathbf{Z}_j \mathbf{w}_j + \mathbf{c}_j{}'\mathbf{c}_j \mathbf{w}_j{}'\mathbf{Z}_j{}'\mathbf{Z}_j \mathbf{w}_j + \mathbf{w}_j{}'\mathbf{Z}_j{}'\mathbf{Z}_j \mathbf{w}_j \\ &\quad -2\left(\alpha_j \mathbf{w}_j{}'\mathbf{Z}_j{}'\mathbf{f}_j + (1-\alpha_j)\mathbf{w}_j{}'\mathbf{Z}_j{}'\mathbf{f}_j + \mathbf{w}_j{}'\mathbf{Z}_j{}'\mathbf{Z}_j \mathbf{c}_j + \mathbf{w}_j{}'\mathbf{Z}_j{}'\boldsymbol{\Phi}_j \mathbf{b}_j\right) \\ &\quad +(1-\alpha_j)\mathbf{f}_j{}'\mathbf{f}_j + \mathbf{b}_j{}'\boldsymbol{\Phi}_j{}'\boldsymbol{\Phi}_j \mathbf{b}_j + \mathrm{trace}(\alpha_j \mathbf{Z}_j{}'\mathbf{Z}_j + \mathbf{Z}_j{}'\mathbf{Z}_j), \qquad (14)\end{aligned}$$

subject to $\boldsymbol{\gamma}_j'\boldsymbol{\gamma}_j = \mathbf{w}_j'\mathbf{Z}_j'\mathbf{Z}_j \mathbf{w}_j = 1$. Note that in (14), both $\mathbf{f}_j$ and $\boldsymbol{\Phi}_j$ do not involve $\mathbf{w}_j$ because $\boldsymbol{\gamma}_j$ is not related to itself. When $\alpha_j = 1$ (i.e., Mode A), minimizing (14) subject to $\mathbf{w}_j'\mathbf{Z}_j'\mathbf{Z}_j \mathbf{w}_j = 1$ is equivalent to minimizing

$$\phi_j^3(\mathbf{w}_j) = \mathbf{w}_j{}'\mathbf{f}_j{}'\mathbf{f}_j \mathbf{w}_j - 2\mathbf{w}_j{}'(\mathbf{Z}_j{}'\mathbf{f}_j + \mathbf{Z}_j{}'\mathbf{Z}_j \mathbf{c}_j + \mathbf{Z}_j{}'\boldsymbol{\Phi}_j \mathbf{b}_j), \qquad (15)$$

subject to $\mathbf{w}_j'\mathbf{Z}_j'\mathbf{Z}_j\mathbf{w}_j = 1$. This quadratic constrained minimization problem cannot be solved analytically. Instead, we minimize (15) iteratively via an interior point algorithm (e.g., Boyd, Boyd, & Vandenberghe, 2004, p. 143), which is efficient for such a problem and implemented into various nonlinear programming solvers, e.g., fmincon in MATLAB and ipop in the R package kernlab (Karatzoglou, Smola, Hornik, & Zeileis, 2004). When $\alpha_j = 0$ (i.e., Mode B), minimizing (14) reduces to maximizing

$$\begin{aligned}\phi_j^4(\mathbf{w}_j) &= \mathbf{w}_j{}'(\mathbf{Z}_j{}'\mathbf{f}_j + \mathbf{Z}_j{}'\mathbf{Z}_j\mathbf{c}_j + \mathbf{Z}_j{}'\mathbf{\Phi}_j\mathbf{b}_j) \\ &= \mathbf{w}_j{}'(\mathbf{Z}_j{}'\mathbf{Z}_j)^{1/2}(\mathbf{Z}_j{}'\mathbf{Z}_j)^{-1/2}(\mathbf{Z}_j{}'\mathbf{f}_j + \mathbf{Z}_j{}'\mathbf{Z}_j\mathbf{c}_j + \mathbf{Z}_j{}'\mathbf{\Phi}_j\mathbf{b}_j) \\ &= \mathbf{v}_j{}'\mathbf{h}_j, \end{aligned} \tag{16}$$

subject to $\mathbf{w}_j'\mathbf{Z}_j'\mathbf{Z}_j\mathbf{w}_j = 1$ or equivalently $\mathbf{v}_j'\mathbf{v}_j = 1$, where $\mathbf{v}_j = (\mathbf{Z}_j{}'\mathbf{Z}_j)^{1/2}\mathbf{w}_j$, and $\mathbf{h}_j = (\mathbf{Z}_j{}'\mathbf{Z}_j)^{-1/2}(\mathbf{Z}_j{}'\mathbf{f}_j + \mathbf{Z}_j{}'\mathbf{Z}_j\mathbf{c}_j + \mathbf{Z}_j{}'\mathbf{\Phi}_j\mathbf{b}_j)$. This maximization problem can be solved as follows: Let $\mathbf{u}_j$ denote the first left singular vector of $\mathbf{h}_j$. Then, $\hat{\mathbf{w}}_j = (\mathbf{Z}_j{}'\mathbf{Z}_j)^{-1/2}\mathbf{u}_j$ (ten Berge, 1993, p. 29). Under both modes, once $\hat{\mathbf{w}}_j$ is obtained, $\boldsymbol{\gamma}_j$ is subsequently updated by $\boldsymbol{\gamma}_j = \mathbf{Z}_j\hat{\mathbf{w}}_j$.

Step 3 We update $\mathbf{c}_j$ with the other parameters fixed. This is equivalent to minimizing

$$\phi_j^5(\mathbf{c}_j) = \mathrm{SS}(\mathbf{Z}_j - \boldsymbol{\gamma}_j\mathbf{c}_j{}'). \tag{17}$$

The least squares estimate of $\mathbf{c}_j$ is obtained as

$$\hat{\mathbf{c}}_j = \mathbf{Z}_j{}'\boldsymbol{\gamma}_j. \tag{18}$$

Step 4 We update $\mathbf{b}_j$ with the other parameters fixed. This is equivalent to minimizing

$$\phi_j^6(\mathbf{b}_j) = \mathrm{SS}(\boldsymbol{\gamma}_j - \mathbf{\Phi}_j\mathbf{b}_j). \tag{19}$$

The least squares estimate of $\mathbf{b}_j$ is obtained as

$$\hat{\mathbf{b}}_j = (\mathbf{\Phi}_j{}'\mathbf{\Phi}_j)^{-1}\mathbf{\Phi}_j{}'\boldsymbol{\gamma}_j. \tag{20}$$

We repeat the above steps until convergence, i.e., the difference in the values of (11) between the previous and current iterations decreases below a predetermined threshold (e.g., .00001). The ALS algorithm monotonically decreases the value of (11), which is also bounded from below. This algorithm is therefore convergent with respect to (11) (i.e., a sequence of the values of (11) is monotone). Yet, the value of (11) at convergence can be a local minimum. To address the issue of convergence to local minima, we may repeat the algorithm with several sets of random initial values for $\mathbf{w}_j$, producing as many values of (11) as sets of random initial values used. Then, the solutions associated with the smallest value of (11) can be chosen as final ones.

Minimization of (11) does not require any distributional assumption, such as multivariate normality of observed variables. Thus, GLSPM is a distribution-free or nonparametric approach. However, as a trade-off, it cannot estimate the standard errors of parameter estimates based on asymptotic (normal theory) approximations. Instead, GLSPM employs the (nonparametric) bootstrap method (Efron, 1979) to obtain the standard errors or confidence intervals of its parameter estimates.

As in Hwang et al.'s (2015) procedure, in theory, GLSPM can allow for a compromise between Mode A and Mode B by taking any value of α_j between 0 and 1. Nevertheless, in practice, it is not clear what this compromise means substantively, when it can be useful, and how the value of α_j can be determined. Thus, we recommend applying GLSPM only under the two conventional modes.

GLSPM is a full-information method in that it estimates all parameters by optimizing a single least squares criterion consistently. This is comparable to GSCA (Hwang & Takane, 2004, 2014), which is also a full-information method for component-based SEM. There is a technical connection between GLSPM and GSCA. Let $\mathbf{Z} = [\mathbf{Z}_1, \ldots, \mathbf{Z}_J]$ denote an N by P matrix of all J blocks of observed variables. Let $\mathbf{W} = \text{diag}[\mathbf{w}_1, \ldots, \mathbf{w}_J]$ denote a P by J matrix consisting of all block-wise vectors of component weights as diagonal elements. Let $\boldsymbol{\Gamma} = \mathbf{ZW} = [\boldsymbol{\gamma}_1, \ldots, \boldsymbol{\gamma}_J]$ denote an N by J matrix consisting of all components. Let $\mathbf{d}_j$ be a J by 1 vector whose elements are all zero except the jth element being unity, so that $\boldsymbol{\gamma}_j = \mathbf{ZWd}_j$. Let $\mathbf{I}_j$ denote a P by P_j row-wise block matrix in which the jth block is the identify matrix of P_j, whereas the other blocks are zero matrices, so that $\mathbf{Z}_j = \mathbf{ZI}_j$. Let $\boldsymbol{\varepsilon}_j$ denote a J by 1 vector consisting of Q_j inner weights (in $\mathbf{e}_j$) for the Q_j components connected to $\boldsymbol{\gamma}_j$ and of $J - Q_j$ zeros for the remaining unconnected components, so that $\mathbf{f}_j = \boldsymbol{\Gamma}_j \mathbf{e}_j = \boldsymbol{\Gamma}\boldsymbol{\varepsilon}_j$. Let $\boldsymbol{\beta}_j$ denote a J by 1 vector consisting of S_j path coefficients relating S_j independent components to $\boldsymbol{\gamma}_j$ and of $J - S_j$ zeros for the remaining components, so that $\boldsymbol{\Gamma}_j\mathbf{b}_j = \boldsymbol{\Gamma}\boldsymbol{\beta}_j$.

Under both modes, minimizing (11) is equivalent to minimizing

$$\phi^*(\mathbf{e}, \mathbf{w}, \mathbf{c}, \mathbf{b}) = \sum_{j=1}^{J} \text{SS}(\mathbf{ZV}_j - \mathbf{ZWA}_j). \tag{21}$$

Specifically, under Mode A or $\alpha_j = 1$, minimizing (11) is equivalent to minimizing

$$\begin{aligned}
\phi^*(\mathbf{e}, \mathbf{w}, \mathbf{c}, \mathbf{b}) &= \sum_{j=1}^{J} \text{SS}(\mathbf{Z}_j - \mathbf{f}_j\mathbf{w}_j{}') + \text{SS}(\mathbf{Z}_j - \boldsymbol{\gamma}_j\mathbf{c}_j{}') + \text{SS}(\boldsymbol{\gamma}_j - \boldsymbol{\Gamma}_j\mathbf{b}_j) \\
&= \sum_{j=1}^{J} \text{SS}\left(\left[\mathbf{Z}_j, \mathbf{Z}_j, \boldsymbol{\gamma}_j\right] - \left[\mathbf{f}_j\mathbf{w}_j{}', \boldsymbol{\gamma}_j\mathbf{c}_j{}', \boldsymbol{\Gamma}_j\mathbf{b}_j\right]\right) \\
&= \sum_{j=1}^{J} \text{SS}\left(\mathbf{Z}\left[\mathbf{I}_j, \mathbf{I}_j, \mathbf{Wd}_j\right] - \mathbf{ZW}\left[\boldsymbol{\varepsilon}_j\mathbf{w}_j{}', \mathbf{d}_j\mathbf{c}_j{}', \boldsymbol{\beta}_j\right]\right) \\
&= \sum_{j=1}^{J} \text{SS}\left(\mathbf{ZV}_j - \mathbf{ZWA}_j\right),
\end{aligned} \tag{22}$$

where $\mathbf{V}_j = [\mathbf{I}_j, \mathbf{I}_j, \mathbf{Wd}_j]$ and $\mathbf{A}_j = [\boldsymbol{\varepsilon}_j\, \mathbf{w}_j', \mathbf{d}_j\mathbf{c}_j', \boldsymbol{\beta}_j]$. Similarly, under Mode B or $\alpha_j = 0$, minimizing (11) is equivalent to minimizing (21), where $\mathbf{V}_j = [\mathbf{I}_j, \mathbf{W}\boldsymbol{\varepsilon}_j, \mathbf{Wd}_j]$ and $\mathbf{A}_j = [\mathbf{c}_j', \mathbf{d}_j, \boldsymbol{\beta}_j]$. The criterion (21) is essentially in the same form of the GSCA criterion (e.g., see Hwang & Takane, 2014, p. 22). A major difference is that (21) is a block-wise criterion for estimating the parameters in $\mathbf{V}_j$ and $\mathbf{A}_j$ for each block. This indicates that GLSPM can be seen as a block-wise special case of GSCA from algorithmic perspectives.

4. Simulated Data Analysis

We conducted a simulation study to evaluate parameter recovery of GLSPM, as compared to existing limited-information PLSPM methods. The existing methods applied Lohmöller's (1989) algorithm, Wold's (1985) algorithm (Hanafi's (2007) algorithm), Hwang et al.'s 2015 procedure, and RGCCA for estimating component weights (and components) in the first stage and then applied a series of least squares regression analyses for estimating loadings and path coefficients as shown in (9) and (10). We called these limited-information methods Lohmöller's PLSPM, Wold's PLSPM, Hwang et al.'s PLSPM, and RGCCA-PLSPM.

We specified a component-based structural equation model, where four independent components influenced two dependent components. Each component was associated with four observed variables. Figure 2 displays the specified model. In the model, we determined component weights ($\mathbf{w}_j$) and loadings ($\mathbf{c}_j$) per component as follows: for Mode A, $\mathbf{w}_j = [.3030; .3535; .3535; .4040]$ and $\mathbf{c}_j = [.6; .7; .7; .8]$, and for Mode B, $\mathbf{w}_j = [.4179; .3582; .3582; .2985]$ and $\mathbf{c}_j = [.6901; .6897; .6897; .7286]$. We considered three experimental factors: mode (Mode A and Mode B), sample size ($N = 100$, 200, 500, and 1000), and the correlation between the independent components ($r = 0$, .2, and .4). This correlation remained the same for each pair of the independent components. For each combination of the levels of the experimental factors, we generated 1000 random samples from a multivariate normal distribution with zero means and the covariance matrix derived from the model specification, using the same data generation procedures in Cho and Choi (2020) and Cho, Jung, and Hwang (2019).

We applied GLSPM and the existing PLSPM methods to fit the model to each sample. When RGCCA-PLSPM was used for the case of Mode A, we applied RGCCA's New Mode A and standardized its components before estimating loadings and path coefficients in the second stage. We also applied Wold's algorithm to the case of Mode A, although his algorithm is monotonically convergent only under Mode B (Hanafi, 2007). We considered the centroid, factorial, and path weighting schemes for Lohmöller's algorithm, whereas we applied the centroid and factorial schemes for Wold's algorithm and RGCCA because no explicit functional form of component covariances or correlations (i.e., $g(\cdot)$ in (8)) for the path weighting scheme is available for Wold's algorithm and RGCCA. For Hwang et al.'s procedure and GLSPM, no scheme needs to be determined before estimation because both methods estimate inner weights by minimizing a least squares optimization criterion consistently (e.g., Step 1 in the ALS algorithm for GLSPM). We did not consider the Horst scheme because our specified model included negatively correlated components, as displayed in Fig. 2. We used the R package RGCCA (version 2.1.2) (Tenenhaus & Guillemot, 2017) for RGCCA, Dr. Arthur Tenenhaus's R code for Wold's algorithm, and wrote MATLAB codes for the other procedures. We used the same initial values for $\mathbf{w}_j$ per sample for all the methods except RGCCA, i.e., $\mathbf{w}_j = \mathbf{1}_j/(\mathbf{1}_j/\mathbf{S}_j\mathbf{1}_j)^{1/2}$, where $\mathbf{1}_j$ is a vector of P_j ones, and $\mathbf{S}_j$ is the correlation matrix of P_j observed variables for the jth component. As the current version of the RGCCA package does not permit users to choose their own initial values for $\mathbf{w}_j$, we used its default initial values that are the elements of the first right singular vector of P_j observed variables for the jth component.

To evaluate the parameter recovery of the methods, we calculated relative bias (RB), standard deviation (SD), and root mean square error (RMSE) of their parameter estimates, as follows.

$$\mathrm{RB}(\hat{\theta}) = (m(\hat{\theta}) - \theta)/\theta, \tag{23}$$

$$\mathrm{SD}(\hat{\theta}) = (\hat{\theta} - m(\hat{\theta}))^2/B, \tag{24}$$

$$\mathrm{RMSE}(\hat{\theta}) = \sqrt{((\hat{\theta} - \theta)^2)/B}, \tag{25}$$

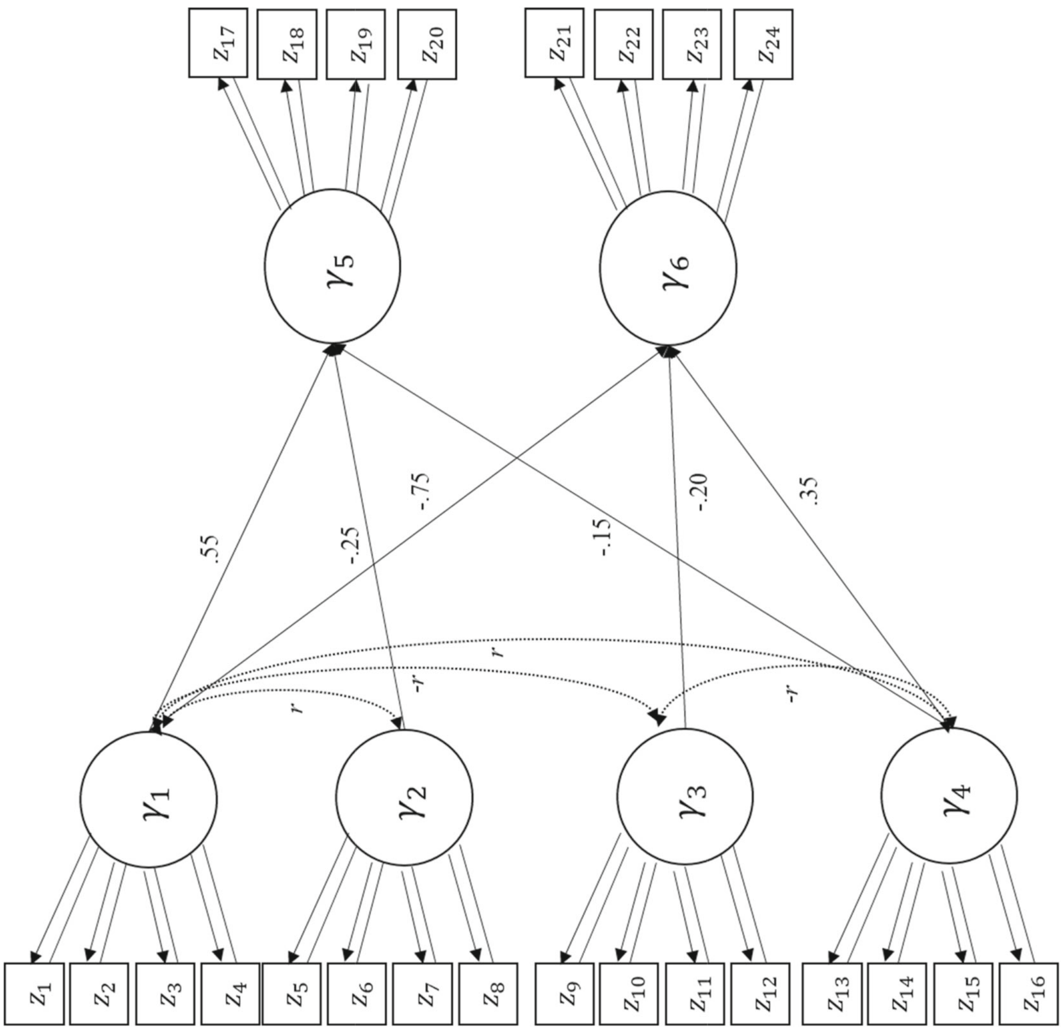

FIGURE 2.
The component-based structural equation model specified for the simulation study. A straight line denotes a component weight, and an arrow indicates a loading or path coefficient. No residual terms are displayed

where θ and $\hat{\theta}$ denote a single parameter and its estimate, respectively, and $m(\hat{\theta})$ is the average of $\hat{\theta}$ over B samples ($B = 1000$ in our study).

To conserve space, we focus here on reporting the average values of these properties of each set of the parameter estimates obtained from the methods under each condition. Note that we used "absolute" relative biases for computing the average RB values. The properties of all individual parameter estimates per condition are provided in Supplementary Materials.

Table 1 presents the average RB, SD, and RMSE values of the estimates of component weights obtained from different PLSPM methods under Mode A. We considered an RB value greater than .1 (or 10%) as indicative of an unacceptable degree of bias (e.g., Bollen, Kirby, Curran, Paxton, & Chen, 2007). On average, GLSPM produced unbiased estimates of component weights in all conditions. Lohmöller's PLSPM resulted in the same weight estimates across all the three different schemes, which were unbiased in most conditions except when $r = .4$ and $N \leq 200$. Wold's PLSPM provided quite similar weight estimates between the centroid and factorial schemes, which were also comparable to those from Lohmöller's PLSPM when $r \leq .2$. However, when $r = .4$, Wold's PLSPM tended to yield biased weight estimates regardless of sample size, although the biases tended to decrease with sample size. RGCCA-PLSPM with the centroid and factorial schemes performed identically to the counterparts of Wold's PLSPM. Hwang et al.'s PLSPM resulted in unbiased weight estimates when $r \leq .2$. However, their method produced biased weight estimates when $r = .4$ unless sample size was large ($N = 1000$).

The weight estimates from GLSPM had much smaller SD values than those from the other limited-information methods in all conditions. The SD values of the weight estimates from all the methods tended to decrease with sample size. However, the SD values of the limited-information methods tended to increase with the correlation between the independent components, whereas those of GPLPM were not be affected by the component correlation. The SD values of the weight estimates from the limited-information methods were quite similar in all conditions. Consequently, in general, the component weight estimates from GLSPM always showed smaller RMSE values than those from the limited-information PLSPM methods in all conditions. The RMSE values of the weight estimates of all the methods decreased with sample size. The RMSE values under the limited-information methods were generally similar across the conditions and increased when the degree of component correlation increased and/or sample size decreased.

Table 2 provides the average RB, SD, and RMSE values of the loading estimates obtained from all the methods under Mode A. These properties of the loading estimates showed similar patterns to those of the weight estimates in Table 1. That is, GLSPM's loading estimates were unbiased and had smaller SD and RMSE values than the other methods' loading estimates in all conditions, although the SD and RMSE values of GLSPM's loading estimates became somewhat larger than those of its weight estimates. The loading estimates of the other limited-information methods generally had similar RB, SD, and RMSE patterns to their corresponding weight estimates.

Table 3 exhibits the average values of the same properties of the path coefficient estimates from the methods under Mode A. On average, GLSPM yielded unbiased estimates of the path coefficients in all conditions. The other limited-information methods also resulted in unbiased estimates of the path coefficients in most conditions except when $r = .4$. When $r = .4$, Lohmöller's PLSPM tended to yield slightly less biased estimates than the other limited-information methods in all sample sizes. This method provided quite similar RB values across the three schemes. Wold's PLSPM and RGCCA-PLSPM yielded identical RB values under the same scheme. When $r \leq .2$, the SD and RMSE values of GLSPM's path coefficient estimates were similar to those of the other methods' estimates across the conditions. When $r = .4$, however, the SD and RMSE values GLSPM's estimates were consistently smaller than those of the other methods' estimates in all sample sizes.

Tables 4 and 5 show the average RB, SD, and RMSE values of the estimates of component weights and loadings obtained from the methods under Mode B. Overall, all the methods performed comparably in recovering both weights and loadings. Specifically, when $r \leq 0.2$, they generally provided unbiased estimates of the parameters except when sample size was relatively small ($N = 100$ or 200). On the other hand, when $r = 0.4$, all the methods yielded biased estimates of the weights and loadings regardless of sample size, although Wold's PLSPM and RGCCA-PLSPM tended to result in slightly more biased estimates under both schemes. The SD

TABLE 1.
Average relative biases (RBs), standard deviations (SDs), and root mean square errors (RMSEs) of component weights estimated from different PLSPM methods under Mode A

	r	N	GLS	PLS^{L}(c)	PLS^{L}(f)	PLS^{L}(p)	PLS^{W}(c)	PLS^{W}(f)	RGCCA(c)	RGCCA(f)	PLS^{H}
RB	0	100	0.00	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.04
		200	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		500	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		1000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	.2	100	0.00	0.07	0.07	0.07	0.10	0.10	0.10	0.10	0.09
		200	0.00	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.05
		500	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1000	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	.4	100	0.00	0.11	0.12	0.12	0.17	0.19	0.17	0.19	0.15
		200	0.00	0.10	0.11	0.11	0.15	0.16	0.15	0.16	0.14
		500	0.00	0.08	0.09	0.09	0.13	0.13	0.13	0.14	0.11
		1000	0.00	0.07	0.07	0.07	0.11	0.11	0.11	0.11	0.09
SD	0	100	0.04	0.12	0.11	0.11	0.12	0.12	0.12	0.12	0.12
		200	0.03	0.08	0.07	0.07	0.08	0.08	0.08	0.08	0.07
		500	0.02	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
		1000	0.01	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	.2	100	0.04	0.16	0.16	0.16	0.17	0.17	0.17	0.17	0.17
		200	0.03	0.13	0.13	0.12	0.13	0.13	0.13	0.13	0.13
		500	0.02	0.08	0.07	0.07	0.08	0.07	0.08	0.07	0.07
		1000	0.01	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
	.4	100	0.04	0.21	0.21	0.21	0.21	0.22	0.22	0.22	0.22
		200	0.03	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19
		500	0.02	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
		1000	0.01	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14
RMSE	0	100	0.04	0.12	0.11	0.11	0.12	0.12	0.12	0.12	0.12
		200	0.03	0.08	0.07	0.07	0.08	0.08	0.08	0.08	0.07
		500	0.02	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
		1000	0.01	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	.2	100	0.04	0.17	0.17	0.17	0.18	0.18	0.18	0.18	0.17
		200	0.03	0.13	0.13	0.13	0.14	0.13	0.14	0.13	0.13
		500	0.02	0.08	0.07	0.07	0.08	0.07	0.08	0.07	0.07
		1000	0.01	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
	.4	100	0.04	0.21	0.22	0.21	0.23	0.23	0.23	0.23	0.23
		200	0.03	0.19	0.19	0.19	0.20	0.20	0.20	0.21	0.20
		500	0.02	0.16	0.16	0.16	0.17	0.17	0.17	0.17	0.17
		1000	0.01	0.14	0.14	0.14	0.15	0.15	0.15	0.15	0.15

GLS = global least squares path modeling, PLS^{L}(c) = Lohmöller's PLSPM with the centroid scheme, PLS^{L}(f) = Lohmöller's PLSPM with the factorial scheme, PLS^{L}(p) = Lohmöller's PLSPM with the path weighting scheme, PLS^{W}(c) = Wold's PLSPM with the centroid scheme, PLS^{W}(f) = Wold's PLSPM with the factorial scheme, RGCCA(c) = RGCCA-PLSPM with the centroid scheme, RGCCA(f) = RGCCA-PLSPM with the factorial scheme, and PLS^{H} = Hwang et al.'s PLSPM values of the estimates from all the methods appeared similar across the conditions and tended to decrease when sample size became larger and/or the degree of component correlation was smaller. Moreover, the RMSE values of the weight and loading estimates from the methods were generally similar in all conditions. Again, the RMSE values tended to decrease when sample size increased

TABLE 2.
Average relative biases (RBs), standard deviations (SDs), and root mean square errors (RMSEs) of loadings estimated from different PLSPM methods under Mode A

	r	N	GLS	PLS^L(c)	PLS^L(f)	PLS^L(p)	PLS^W(c)	PLS^W(f)	RGCCA(c)	RGCCA(f)	PLS^H
RB	0	100	0.01	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.04
		200	0.00	0.01	0.01	0.01	0.02	0.01	0.02	0.01	0.01
		500	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		1000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	.2	100	0.01	0.07	0.07	0.07	0.09	0.10	0.09	0.10	0.09
		200	0.00	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.05
		500	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1000	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	.4	100	0.01	0.11	0.13	0.12	0.15	0.16	0.15	0.16	0.16
		200	0.00	0.10	0.11	0.11	0.13	0.14	0.13	0.14	0.14
		500	0.00	0.08	0.09	0.09	0.11	0.11	0.11	0.12	0.11
		1000	0.00	0.07	0.07	0.07	0.09	0.09	0.09	0.09	0.09
SD	0	100	0.07	0.12	0.12	0.12	0.13	0.14	0.14	0.14	0.13
		200	0.05	0.07	0.07	0.07	0.08	0.08	0.08	0.08	0.07
		500	0.03	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
		1000	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	.2	100	0.07	0.16	0.16	0.16	0.19	0.19	0.19	0.19	0.18
		200	0.05	0.12	0.12	0.12	0.14	0.14	0.14	0.14	0.13
		500	0.03	0.06	0.06	0.06	0.07	0.07	0.07	0.07	0.07
		1000	0.02	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
	.4	100	0.07	0.19	0.20	0.20	0.23	0.23	0.23	0.23	0.23
		200	0.05	0.17	0.17	0.17	0.19	0.20	0.19	0.20	0.20
		500	0.03	0.14	0.14	0.14	0.16	0.16	0.16	0.17	0.17
		1000	0.02	0.12	0.12	0.12	0.15	0.14	0.15	0.14	0.14
RMSE	0	100	0.07	0.12	0.12	0.12	0.14	0.14	0.14	0.14	0.13
		200	0.05	0.07	0.07	0.07	0.08	0.08	0.08	0.08	0.07
		500	0.03	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
		1000	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	.2	100	0.07	0.17	0.17	0.17	0.20	0.21	0.20	0.21	0.20
		200	0.05	0.12	0.12	0.12	0.14	0.14	0.14	0.14	0.14
		500	0.03	0.07	0.06	0.06	0.07	0.07	0.07	0.07	0.07
		1000	0.02	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
	.4	100	0.07	0.21	0.22	0.22	0.25	0.26	0.26	0.27	0.26
		200	0.05	0.18	0.19	0.19	0.22	0.23	0.22	0.23	0.22
		500	0.03	0.15	0.16	0.15	0.18	0.19	0.18	0.19	0.19
		1000	0.02	0.13	0.13	0.13	0.16	0.16	0.16	0.16	0.16

GLS = global least squares path modeling, PLS^L(c) = Lohmöller's PLSPM with the centroid scheme, PLS^L(f) = Lohmöller's PLSPM with the factorial scheme, PLS^L(p) = Lohmöller's PLSPM with the path weighting scheme, PLS^W(c) = Wold's PLSPM with the centroid scheme, PLS^W(f) = Wold's PLSPM with the factorial scheme, RGCCA(c) = RGCCA-PLSPM with the centroid scheme, RGCCA(f) = RGCCA-PLSPM with the factorial scheme, and PLS^H = Hwang et al.'s PLSPM

and/or the level of component correlation decreased. Wold's PLSPM and RGCCA-PLSPM performed equally in recovering both weights and loadings under the same scheme.

Table 6 presents the average RB, SD, and RMSE values of the path coefficient estimates obtained from the methods under Mode B. The overall patterns of the properties of the path coefficient estimates were similar to those of the weight and loading estimates. That is, on average,

TABLE 3.
Average relative biases (RBs), standard deviations (SDs), and root mean square errors (RMSEs) of path coefficients estimated from different PLSPM methods under Mode A

	r	N	GLS	PLS^L(c)	PLS^L(f)	PLS^L(p)	PLS^W(c)	PLS^W(f)	RGCCA(c)	RGCCA(f)	PLS^H
RB	0	100	0.01	0.04	0.02	0.02	0.03	0.03	0.03	0.03	0.03
		200	0.01	0.02	0.01	0.01	0.02	0.01	0.02	0.01	0.01
		500	0.00	0.01	0.00	0.00	0.01	0.00	0.01	0.00	0.00
		1000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	.2	100	0.01	0.03	0.04	0.04	0.05	0.07	0.06	0.07	0.06
		200	0.01	0.02	0.02	0.02	0.02	0.03	0.02	0.03	0.03
		500	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
		1000	0.00	0.01	0.00	0.00	0.01	0.00	0.01	0.00	0.00
	.4	100	0.01	0.19	0.22	0.19	0.25	0.28	0.26	0.28	0.23
		200	0.01	0.17	0.17	0.16	0.20	0.21	0.20	0.21	0.19
		500	0.00	0.13	0.13	0.13	0.16	0.17	0.17	0.17	0.16
		1000	0.00	0.10	0.11	0.10	0.13	0.13	0.13	0.13	0.12
SD	0	100	0.07	0.07	0.07	0.07	0.07	0.08	0.07	0.08	0.07
		200	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
		500	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
		1000	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	.2	100	0.08	0.09	0.09	0.09	0.10	0.11	0.10	0.11	0.10
		200	0.05	0.06	0.06	0.06	0.07	0.07	0.07	0.07	0.07
		500	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
		1000	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	.4	100	0.09	0.12	0.12	0.12	0.14	0.14	0.14	0.14	0.13
		200	0.06	0.09	0.09	0.09	0.10	0.10	0.10	0.10	0.10
		500	0.04	0.06	0.06	0.06	0.07	0.07	0.07	0.07	0.07
		1000	0.03	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.06
RMSE	0	100	0.07	0.07	0.07	0.07	0.08	0.08	0.08	0.08	0.07
		200	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
		500	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
		1000	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	.2	100	0.08	0.09	0.09	0.09	0.11	0.11	0.11	0.11	0.10
		200	0.05	0.06	0.06	0.06	0.07	0.07	0.07	0.07	0.07
		500	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.04
		1000	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	.4	100	0.09	0.13	0.14	0.14	0.15	0.16	0.16	0.16	0.15
		200	0.06	0.10	0.10	0.10	0.11	0.12	0.11	0.12	0.11
		500	0.04	0.07	0.07	0.07	0.08	0.08	0.08	0.08	0.08
		1000	0.03	0.05	0.06	0.05	0.07	0.07	0.07	0.07	0.06

GLS = global least squares path modeling, PLS^L(c) = Lohmöller's PLSPM with the centroid scheme, PLS^L(f) = Lohmöller's PLSPM with the factorial scheme, PLS^L(p) = Lohmöller's PLSPM with the path weighting scheme, PLS^W(c) = Wold's PLSPM with the centroid scheme, PLS^W(f) = Wold's PLSPM with the factorial scheme, RGCCA(c) = RGCCA-PLSPM with the centroid scheme, RGCCA(f) = RGCCA-PLSPM with the factorial scheme, and PLS^H = Hwang et al.'s PLSPM

the path coefficient estimates from all the methods showed similar levels of bias in many conditions, although Wold's PLSPM and RGCCA-PLSPM tended to produce more biased estimates under both centroid and factorial schemes when $r \geq 0.2$. The SD and RMSE values of the path coefficient estimates generally were similar among the methods, although those under Wold's PLSPM and RGCCA-PLSPM were slightly larger when $N \leq 200$ and/or $r = 0.4$. The RB,

TABLE 4.
Average relative biases (RBs), standard deviations (SDs), and root mean square errors (RMSEs) of component weights estimated from different PLSPM methods under Mode B

	r	N	GLS	PLS^{L}(c)	PLS^{L}(f)	PLS^{L}(p)	PLS^{W}(c)	PLS^{W}(f)	RGCCA(c)	RGCCA(f)	PLS^{H}
RB	0	100	0.11	0.12	0.12	0.12	0.15	0.14	0.15	0.14	0.12
		200	0.06	0.07	0.06	0.06	0.07	0.07	0.07	0.07	0.06
		500	0.02	0.03	0.02	0.02	0.03	0.02	0.03	0.02	0.02
		1000	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	.2	100	0.20	0.21	0.22	0.21	0.27	0.27	0.27	0.27	0.21
		200	0.13	0.15	0.14	0.14	0.18	0.17	0.18	0.17	0.14
		500	0.06	0.07	0.07	0.07	0.08	0.08	0.08	0.08	0.07
		1000	0.03	0.04	0.03	0.03	0.04	0.04	0.04	0.04	0.03
	.4	100	0.28	0.30	0.31	0.30	0.38	0.38	0.38	0.38	0.31
		200	0.24	0.25	0.26	0.26	0.31	0.31	0.31	0.31	0.26
		500	0.19	0.20	0.20	0.20	0.25	0.25	0.25	0.25	0.21
		1000	0.16	0.17	0.17	0.17	0.21	0.21	0.21	0.21	0.17
SD	0	100	0.24	0.26	0.25	0.25	0.26	0.25	0.26	0.25	0.25
		200	0.18	0.19	0.18	0.18	0.20	0.18	0.20	0.18	0.18
		500	0.11	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.11
		1000	0.08	0.09	0.08	0.08	0.09	0.08	0.09	0.08	0.08
	.2	100	0.31	0.33	0.32	0.32	0.33	0.32	0.34	0.32	0.32
		200	0.25	0.27	0.26	0.26	0.27	0.26	0.27	0.26	0.25
		500	0.17	0.19	0.18	0.18	0.19	0.18	0.19	0.18	0.17
		1000	0.13	0.14	0.13	0.13	0.14	0.13	0.14	0.13	0.13
	.4	100	0.36	0.39	0.38	0.37	0.39	0.38	0.39	0.38	0.38
		200	0.31	0.34	0.33	0.32	0.34	0.32	0.34	0.32	0.32
		500	0.26	0.28	0.27	0.27	0.28	0.27	0.28	0.27	0.27
		1000	0.23	0.25	0.23	0.23	0.24	0.23	0.24	0.23	0.23
RMSE	0	100	0.24	0.27	0.25	0.25	0.27	0.25	0.27	0.25	0.25
		200	0.18	0.20	0.18	0.18	0.20	0.18	0.20	0.18	0.18
		500	0.11	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.11
		1000	0.08	0.09	0.08	0.08	0.09	0.08	0.09	0.08	0.08
	.2	100	0.32	0.34	0.33	0.33	0.35	0.34	0.35	0.34	0.33
		200	0.25	0.28	0.26	0.26	0.28	0.27	0.28	0.27	0.26
		500	0.17	0.19	0.18	0.18	0.20	0.18	0.20	0.18	0.18
		1000	0.13	0.14	0.13	0.13	0.14	0.13	0.14	0.13	0.13
	.4	100	0.38	0.41	0.40	0.39	0.42	0.41	0.42	0.41	0.40
		200	0.33	0.36	0.35	0.34	0.36	0.35	0.36	0.35	0.34
		500	0.27	0.30	0.29	0.28	0.30	0.29	0.30	0.29	0.28
		1000	0.24	0.25	0.24	0.24	0.26	0.25	0.26	0.25	0.24

GLS = global least squares path modeling, PLS^{L}(c) = Lohmöller's PLSPM with the centroid scheme, PLS^{L}(f) = Lohmöller's PLSPM with the factorial scheme, PLS^{L}(p) = Lohmöller's PLSPM with the path weighting scheme, PLS^{W}(c) = Wold's PLSPM with the centroid scheme, PLS^{W}(f) = Wold's PLSPM with the factorial scheme, RGCCA(c) = RGCCA-PLSPM with the centroid scheme, RGCCA(f) = RGCCA-PLSPM with the factorial scheme, and PLS^{H} = Hwang et al.'s PLSPM

SD, and RMSE values of the path coefficient estimates from all the methods tended to decrease when sample size increased and/or the degree of component correlation decreased. Again, Wold's PLSPM and RGCCA-PLSPM performed equally in recovering path coefficients under the same scheme.

TABLE 5.
Average relative biases (RBs), standard deviations (SDs), and root mean square errors (RMSEs) of loadings estimated from different PLSPM methods under Mode B

	r	N	GLS	PLS^L(c)	PLS^L(f)	PLS^L(p)	PLS^W(c)	PLS^W(f)	RGCCA(c)	RGCCA(f)	PLS^H
RB	0	100	0.12	0.13	0.12	0.12	0.15	0.14	0.15	0.14	0.12
		200	0.06	0.07	0.06	0.06	0.08	0.07	0.08	0.07	0.06
		500	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
		1000	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	.2	100	0.20	0.22	0.22	0.22	0.27	0.27	0.27	0.27	0.22
		200	0.13	0.15	0.14	0.14	0.18	0.17	0.18	0.17	0.14
		500	0.07	0.07	0.07	0.07	0.08	0.08	0.08	0.08	0.07
		1000	0.03	0.04	0.03	0.03	0.04	0.04	0.04	0.04	0.03
	.4	100	0.29	0.31	0.32	0.31	0.37	0.38	0.38	0.37	0.32
		200	0.24	0.26	0.26	0.26	0.31	0.31	0.31	0.31	0.26
		500	0.19	0.20	0.21	0.20	0.25	0.25	0.25	0.25	0.21
		1000	0.16	0.17	0.17	0.17	0.21	0.21	0.21	0.21	0.17
SD	0	100	0.18	0.19	0.18	0.18	0.22	0.21	0.22	0.21	0.19
		200	0.13	0.14	0.13	0.13	0.15	0.14	0.15	0.14	0.13
		500	0.08	0.09	0.08	0.08	0.09	0.08	0.09	0.08	0.08
		1000	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
	.2	100	0.23	0.25	0.24	0.24	0.27	0.27	0.27	0.27	0.24
		200	0.18	0.20	0.19	0.19	0.22	0.21	0.22	0.21	0.19
		500	0.12	0.14	0.13	0.13	0.15	0.14	0.15	0.14	0.13
		1000	0.09	0.10	0.09	0.09	0.10	0.09	0.10	0.09	0.09
	.4	100	0.27	0.29	0.29	0.28	0.32	0.31	0.32	0.31	0.29
		200	0.23	0.25	0.24	0.24	0.27	0.26	0.27	0.26	0.24
		500	0.19	0.21	0.20	0.20	0.22	0.22	0.22	0.22	0.20
		1000	0.17	0.18	0.17	0.17	0.20	0.19	0.20	0.19	0.18
RMSE	0	100	0.20	0.21	0.20	0.20	0.24	0.23	0.24	0.23	0.21
		200	0.14	0.15	0.14	0.14	0.16	0.15	0.16	0.15	0.14
		500	0.08	0.09	0.08	0.08	0.09	0.08	0.09	0.08	0.08
		1000	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
	.2	100	0.27	0.29	0.29	0.29	0.34	0.33	0.34	0.33	0.29
		200	0.21	0.23	0.22	0.22	0.26	0.25	0.26	0.25	0.22
		500	0.13	0.15	0.14	0.14	0.16	0.15	0.16	0.15	0.13
		1000	0.09	0.10	0.09	0.09	0.11	0.10	0.11	0.10	0.09
	.4	100	0.34	0.36	0.37	0.36	0.42	0.41	0.42	0.41	0.37
		200	0.29	0.31	0.31	0.31	0.36	0.35	0.36	0.35	0.31
		500	0.24	0.26	0.25	0.25	0.29	0.29	0.29	0.29	0.26
		1000	0.21	0.22	0.21	0.21	0.25	0.25	0.25	0.25	0.22

GLS = global least squares path modeling, PLS^L(c) = Lohmöller's PLSPM with the centroid scheme, PLS^L(f) = Lohmöller's PLSPM with the factorial scheme, PLS^L(p) = Lohmöller's PLSPM with the path weighting scheme, PLS^W(c) = Wold's PLSPM with the centroid scheme, PLS^W(f) = Wold's PLSPM with the factorial scheme, RGCCA(c) = RGCCA-PLSPM with the centroid scheme, RGCCA(f) = RGCCA-PLSPM with the factorial scheme, and PLS^H = Hwang et al.'s PLSPM

In sum, our study showed that under Mode A, GLSPM generally recovered component weights and loadings better than the existing limited-information methods. It produced unbiased estimates of these parameters in all conditions, whereas the other methods resulted in biased estimates in some conditions (e.g., sample size was small and/or the degree of component correlation was moderately sizeable). Moreover, GLSPM's weight and loading estimates always had smaller

TABLE 6.
Average relative biases (RBs), standard deviations (SDs), and root mean square errors (RMSEs) of path coefficients estimated from different PLSPM methods under Mode B

	r	N	GLS	PLS^L(c)	PLS^L(f)	PLS^L(p)	PLS^W(c)	PLS^W(f)	RGCCA(c)	RGCCA(f)	PLS^H
RB	0	100	0.08	0.08	0.06	0.06	0.08	0.07	0.08	0.07	0.09
		200	0.04	0.05	0.04	0.04	0.05	0.04	0.05	0.04	0.05
		500	0.02	0.02	0.01	0.01	0.02	0.01	0.02	0.01	0.02
		1000	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	.2	100	0.15	0.14	0.16	0.15	0.24	0.26	0.25	0.26	0.18
		200	0.10	0.09	0.10	0.09	0.14	0.15	0.14	0.15	0.11
		500	0.05	0.04	0.05	0.04	0.05	0.06	0.05	0.06	0.05
		1000	0.03	0.02	0.03	0.02	0.02	0.03	0.02	0.03	0.03
	.4	100	0.42	0.48	0.49	0.45	0.58	0.59	0.59	0.59	0.46
		200	0.34	0.39	0.40	0.37	0.47	0.47	0.47	0.47	0.37
		500	0.27	0.30	0.29	0.28	0.35	0.35	0.35	0.35	0.29
		1000	0.23	0.25	0.24	0.23	0.29	0.28	0.29	0.28	0.24
SD	0	100	0.08	0.07	0.07	0.08	0.10	0.09	0.10	0.09	0.08
		200	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
		500	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
		1000	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	.2	100	0.10	0.11	0.11	0.11	0.14	0.14	0.14	0.14	0.11
		200	0.07	0.07	0.07	0.07	0.09	0.09	0.09	0.09	0.07
		500	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.04
		1000	0.02	0.03	0.02	0.02	0.03	0.03	0.03	0.03	0.02
	.4	100	0.14	0.14	0.14	0.14	0.18	0.17	0.18	0.17	0.15
		200	0.09	0.10	0.10	0.10	0.12	0.12	0.12	0.12	0.10
		500	0.06	0.06	0.07	0.07	0.08	0.08	0.08	0.08	0.07
		1000	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.05
RMSE	0	100	0.08	0.08	0.08	0.08	0.10	0.10	0.10	0.10	0.09
		200	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.05
		500	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
		1000	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	.2	100	0.11	0.13	0.12	0.12	0.17	0.16	0.17	0.16	0.13
		200	0.07	0.08	0.08	0.08	0.11	0.10	0.11	0.10	0.08
		500	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.04
		1000	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	.4	100	0.18	0.20	0.20	0.19	0.24	0.24	0.25	0.24	0.20
		200	0.13	0.15	0.15	0.14	0.18	0.18	0.18	0.18	0.15
		500	0.10	0.11	0.11	0.10	0.13	0.13	0.13	0.13	0.11
		1000	0.08	0.08	0.08	0.08	0.10	0.10	0.10	0.10	0.09

GLS = global least squares path modeling, PLS^L(c) = Lohmöller's PLSPM with the centroid scheme, PLS^L(f) = Lohmöller's PLSPM with the factorial scheme, PLS^L(p) = Lohmöller's PLSPM with the path weighting scheme, PLS^W(c) = Wold's PLSPM with the centroid scheme, PLS^W(f) = Wold's PLSPM with the factorial scheme, RGCCA(c) = RGCCA-PLSPM with the centroid scheme, RGCCA(f) = RGCCA-PLSPM with the factorial scheme, and PLS^H = Hwang et al.'s PLSPM

SD and RMSE values than the other methods' counterparts. GLSPM recovered path coefficients better than or equally to the other methods. It resulted in unbiased estimates of path coefficients in all conditions, whereas the existing methods yielded unbiased estimates when sample size was large and/or the degree of component correlation was small. The SD and RMSE values of the path coefficient estimates under GLSPM were smaller than those under the other methods when $r = 0.4$. Consequently, under Mode A, GLSPM can be a suitable alternative to the existing PLSPM methods.

On the other hand, under Mode B, GLSPM recovered all the parameters comparably to the existing methods. The estimates of all the methods generally showed similar RB, SD, and RMSE values across the conditions. Thus, in practice, it may be acceptable to employ any of the methods to estimate parameters under Mode B.

All the existing limited-information PLSPM methods generally performed similarly to one another under both modes, although Lohmöller's PLSPM tended to provide somewhat less biased estimates in some conditions (e.g., $r = .4$). Accordingly, there may be little empirical preference among the existing methods. This suggests that although Lohmöller's algorithm does not guarantee monotonic convergence, the algorithm may still be chosen for both Mode A and Mode B. Moreover, under Mode A, it may be acceptable to apply RGCCA's New Mode A to estimate component weights and components in the first estimation stage, and subsequently standardize the components and employ the standardized ones for estimating loadings and path coefficients in the second stage. Wold's PLSPM may also be employed under Mode A, although his algorithm is not monotonically convergent in this case. As expected, under Mode B, Wold's PLSPM and RGCCA-PLSPM with the same scheme performed identically in all conditions, indicating that RGCCA can subsume Wold's algorithm as a special case in this case. This also suggests that using different initial values between RGCCA and the other methods seemed to have little impact on their relative performance. Lastly, choosing a different scheme in Lohmöller's and Wold's algorithms and RGCCA did not lead to substantial differences in the results under both modes.

5. Real Data Analysis

The present example was part of the American customer satisfaction index (ACSI; Fornell, Johnson, Anderson, Cha, & Bryant, 1996) database. The ACSI has been used for measuring cumulative customer satisfaction, which represents the customer's overall evaluation based on her/his purchase and consumption experience with a product or service over time (e.g., Anderson et al. 1994; Fornell, 1992). This example was consumer-level data collected in 2002, which included the responses of 774 consumers to the service units (e.g., police, garbage pickup services, etc.) within the US sector of public administration ($N = 774$).

Figure 3 displays the ACSI model. The ACSI model contains six constructs: customer expectations (CE), perceived quality (PQ), perceived value (PV), customer satisfaction (CS), customer complaints (CC), and customer loyalty (CL). Customer satisfaction is a focal construct and the others are its antecedents and consequences. The structural relationships among these constructs were well derived based on previous theories and studies (Fornell et al., 1996). The model includes a total of 14 observed variables associated with the six constructs, as shown in Table 7. The measures and scales of the observed variables are described in Fornell et al. 1996. Conventionally, PLSPM has been adopted for fitting the ACSI model (Anderson & Fornell, 2000; Fornell et al., 1996; Johnson, Gustafsson, Andreassen, Lervik, & Cha, 2001).

Mode A has been chosen for estimating the components in the ACSI model (e.g., Fornell et al., 1996). Thus, we applied GLSPM and Lohmöller's PLSPM to the data, because as shown in the simulation study, Lohmöller's PLSPM performed similarly to or slightly better than the other existing PLSPM methods. We used the same MATLAB code for GLSPM and SmartPLS (Ringle et

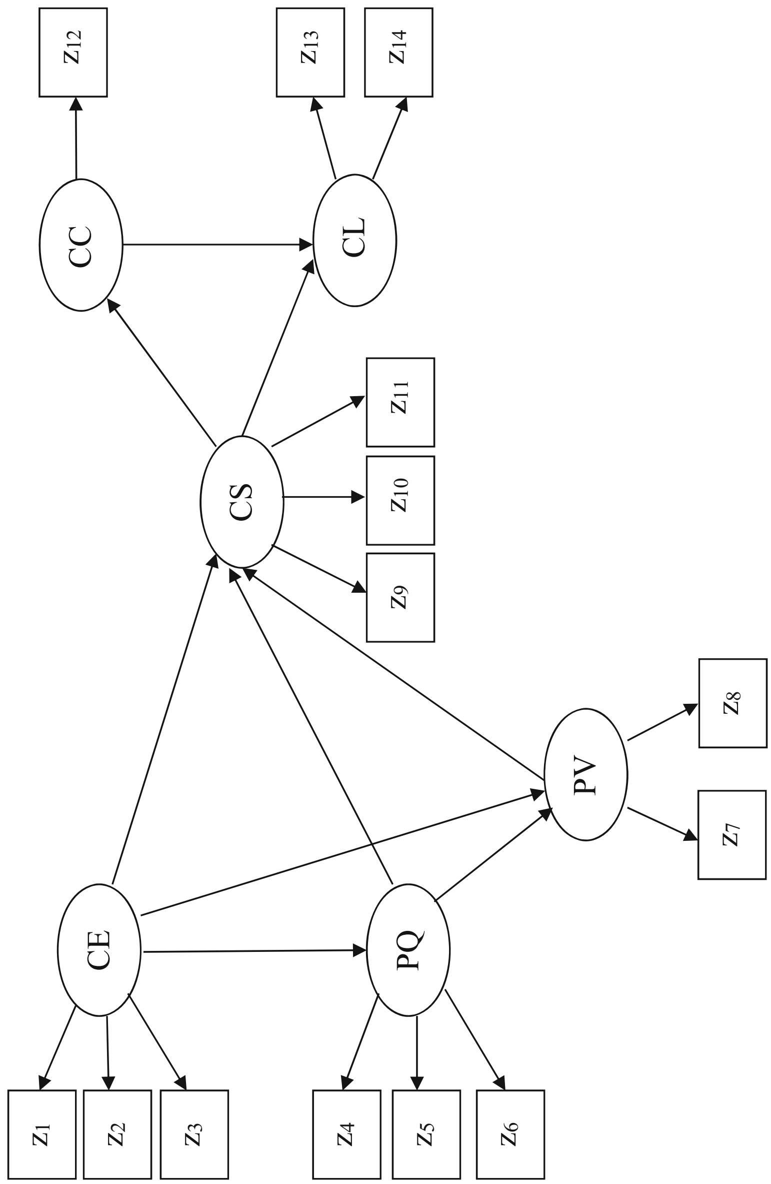

FIGURE 3.
The American customer satisfaction index model. No residual terms are displayed

TABLE 7.
Component weight and loading estimates and their standard errors (SEs) obtained from GLSPM and Lohmöller's PLSPM with the path weighting scheme ($PLSPM^L$) for the ACSI model

Components	Observed variables	GLSPM		$PLSPM^L$	
		Weight estimates (SE)	Loading estimates (SE)	Weight estimates (SE)	Loading estimates (SE)
CE	z_1 = customer expectations about overall quality	0.44 (0.02)	0.86 (0.01)	0.45 (0.06)	0.87 (0.03)
	z_2 = customer expectations about reliability	0.43 (0.02)	0.88 (0.01)	0.43 (0.06)	0.88 (0.04)
	z_3 = customer expectations about customization	0.34 (0.02)	0.73 (0.03)	0.32 (0.06)	0.72 (0.11)
PQ	z_4 = overall quality	0.40 (0.01)	0.93 (0.01)	0.41 (0.02)	0.93 (0.02)
	z_5 = reliability	0.41 (0.01)	0.93 (0.01)	0.40 (0.03)	0.93 (0.02)
	z_6 = customization	0.30 (0.01)	0.80 (0.02)	0.31 (0.02)	0.80 (0.07)
PV	z_7 = price given quality	0.41 (0.02)	0.79 (0.02)	0.42 (0.05)	0.80 (0.08)
	z_8 = quality given price	0.72 (0.02)	0.94 (0.01)	0.71 (0.07)	0.93 (0.01)
CS	z_9 = overall customer satisfaction	0.39 (0.01)	0.94 (0.01)	0.39 (0.01)	0.94 (0.02)
	z_{10} = confirmation of expectations	0.35 (0.01)	0.92 (0.01)	0.34 (0.02)	0.92 (0.03)
	z_{11} = distance to ideal product or service	0.35 (0.01)	0.91 (0.01)	0.35 (0.02)	0.91 (0.03)
CC	z_{12} = formal or informal complaint behavior	1.00 (0)	1.00 (0)	1.00 (0)	1.00 (0)
CL	z_{13} = repurchase intention	0.60 (0.01)	0.95 (0.00)	0.58 (0.03)	0.95 (0.01)
	z_{14} = price tolerance	0.47 (0.01)	0.92 (0.01)	.48 (0.03)	0.93 (0.02)

TABLE 8.
Estimates of path coefficients and their standard errors (SEs) obtained from GLSPM and Lohmöller's PLSPM with the path weighting scheme (PLSPM^{L}) for the ACSI model

	GLSPM	PLSPM^{L}
CE → PQ (b_1)	0.58 (0.03)	0.58 (0.08)
CE → PV (b_2)	0.12 (0.04)	0.12 (0.09)
CE → CS (b_3)	0.03 (0.02)	0.04 (0.06)
PQ → PV (b_4)	0.65 (0.03)	0.65 (0.09)
PQ → CS (b_5)	0.67 (0.03)	0.67 (0.08)
PV → CS (b_6)	0.27 (0.03)	0.27 (0.09)
CS → CC (b_7)	−0.40 (0.04)	−0.40 (0.10)
CS → CL (b_8)	0.58 (0.03)	0.58 (0.11)
CC → CL (b_9)	−0.10(0.04)	−0.10 (0.09)

al., 2005) for Lohmöller's PLSPM. We used the path weighting scheme for Lohmöller's PLSPM, which is recommended over the other schemes (Chin, 1998; Vinzi et al., 2010), and employed 100 bootstrap samples for the estimation of standard errors for both methods.

Table 7 presents the estimates of component weights and loadings obtained from GLSPM and Lohmöller's PLSPM. Both methods resulted in quite similar estimates of the component weights and loadings. This appears to be consistent with that in the simulation study, the two methods generally provided unbiased estimates of these parameters when sample size was large (e.g., $N > 500$). All their estimates were statistically significant and large in size. This indicates that all the observed variables contributed to forming their components and the components in turn explained their observed variables well. Nonetheless, both weights and loading estimates obtained under GLSPM involved consistently much smaller standard errors than those under Lohmöller's PLSPM. This is also consistent with our finding in the simulation study that GLSPM tended to provide more efficient estimates of component weights and loadings than the existing PLSPM methods under Mode A.

Table 8 shows the path coefficient estimates obtained from the two methods. Again, both methods resulted in almost identical path coefficient estimates, leading to the same interpretations. This seems to resonate the result of the simulation study that on average, Lohmöller's PLSPM tended to provide unbiased estimates of path coefficients when sample size was large. In general, the interpretations of the path coefficient estimates were consistent with the relationships hypothesized in the ACSI model. However, the standard errors of the path coefficient estimates obtained under Lohmöller's PLSPM were consistently larger than those under GLSPM. Three path coefficient estimates from Lohmöller's PLSPM turned out to be statistically insignificant. Specifically, customer expectations (CE) had statistically insignificant influences on perceived value (PV) ($b_2 = 0.12$, SE = 0.09) and customer satisfaction (CS) ($b_3 = 0.04$, SE = 0.06), and customer complaints (CC) had a statistically insignificant effect on customer loyalty (CL) ($b_9 = -0.10$, SE = 0.09). On the other hand, under GLSPM, only customer expectations (CE) had a statistically insignificant impact on customer satisfaction (CS) ($b_3 = 0.03$, SE = 0.02). It would be difficult to evaluate which method provided more accurate path coefficient estimates because their parameter values are unknown. Nonetheless, GLSPM tended to provide more reliable estimates of the path coefficients, which also appeared to be more congruent with the relationships hypothesized in the ACSI. This is also consistent with that in the simulation study, the estimates of path coefficients under GLSPM always had smaller standard deviations than those under the other PLSPM methods, when the degree of component correlations was moderately large ($r = 0.4$).

The correlation between CE and PQ was 0.58, which are independent variables for both PV and CS in the structural model. The correlation between CS and CC was 0.4, which are independent variables for CL.

6. Concluding Remarks

We proposed a full-information method for PLSPM, named GLSPM, where a single optimization criterion is minimized via a simple iterative algorithm for estimating all parameters simultaneously. The ALS algorithm for GLSPM is easy to implement and estimates the parameters optimally in the least squares sense. Thus, GLSPM completely addresses the enduring technical issue of PLSPM—the absence of a clear single optimization criterion for estimating all parameters at once under both Mode A and Mode B. Furthermore, it would be of theoretical importance that the optimization criterion for GLSPM can be regarded as a block-wise special case of the criterion for GSCA, contributing to a better understanding of the relation between PLSPM and GSCA from algorithmic perspectives.

Our simulation study shows that under Mode A, GLSPM performed better than or equally to the existing limited-information PLSPM methods in recovering parameters, while under Mode B, it performed similarly to the existing ones. Our real data analysis additionally supported the results of the simulation study under Mode A. Thus, as a whole, GLSPM can serve as an alternative to the existing PLSPM methods because it is theoretically better defined and empirically performs better than or comparably to the existing ones under both modes.

References

Addinsoft, S. (2009). XLSTAT 2009. *Addinsoft*. Paris, France: Addinsoft.

Anderson, E. W., & Fornell, C. (2000). Foundations of the American customer satisfaction index. *Total Quality Management*, *11*(7), 869–882. https://doi.org/10.1080/09544120050135425.

Anderson, E. W., Fornell, C., & Lehmann, D. R. (1994). Customer satisfaction, market share, and profitability: Findings from Sweden. *Journal of Marketing*, *58*(3), 53–66. https://doi.org/10.2307/1252310.

Bollen, K. A., Kirby, J. B., Curran, P. J., Paxton, P. M., & Chen, F. (2007). Latent variable models under misspecification two-stage least squares (2SLS) and maximum likelihood (ML) estimators. *Sociological Methods and Research*, *36*(1), 48–86. https://doi.org/10.1177/0049124107301947.

Boyd, S., Boyd, S. P., & Vandenberghe, L. (2004). *Convex optimization*. Cambridge, England: Cambridge University Press.

Chin, W. W. (1998). The partial least squares approach for structural equation modeling. In G. A. Marcoulides (Ed.), *Modern methods for business research* (pp. 295–336). Mahwah, NJ, US: Erlbaum.

Chin, W. W. (2001). *PLS-Graph User's Guide Version 3.0.* Soft Modeling Inc.

Cho, G., & Choi, J. Y. (2020). An empirical comparison of generalized structured component analysis and partial least squares path modeling under variance-based structural equation models. *Behaviormetrika*, *47*(1), 243–272. https://doi.org/10.1007/s41237-019-00098-0.

Cho, G., Jung, K., & Hwang, H. (2019). Out-of-bag prediction error?: A cross validation index for generalized structured component analysis. *Multivariate Behavioral Research*,. https://doi.org/10.1080/00273171.2018.1540340.

Cho, G., Sarstedt, M., & Hwang, H. (2020). A comparative evaluation of factor- and component-based structural equation modeling methods under (in)consistent model specifications. *Manuscript Submitted for Publication*.

Coolen, H., & de Leeuw, J. (1987). Least squares path analysis with optimal scaling. Paper presented at the fifth international symposium of data analysis and informatics.

de Leeuw, J., Young, F. W., & Takane, Y. (1976). Additive structure in qualitative data: An alternating least squares method with optimal scaling features. *Psychometrika*, *41*(4), 471–503. https://doi.org/10.1007/BF02296971.

Dijkstra, T. K. (2010). Latent variables and indices: Herman Wold's basic design and partial least squares. In V. Esposito Vinzi, W. W. Chin, J. Henseler, & H. Wang (Eds.), *Handbook of Partial Least Squares: Concepts, Methods and Applications* (pp. 23–46). Berlin, Heidelberg: Springer. https://doi.org/10.1007/978-3-540-32827-8_2

Dijkstra, T. K. (2017). A perfect match between a model and a mode. In *Partial least squares path modeling: Basic concepts, methodological issues and applications* (pp. 55–80). Berlin, Germany: Springer-Verlag. https://doi.org/10.1007/978-3-319-64069-3_4

Dijkstra, T. K., & Henseler, J. (2015). Consistent partial least squares path modeling. *MIS Quarterly*, *39*(2), 297–316. https://doi.org/10.25300/misq/2015/39.2.02.

Efron, B. (1979). Bootstrap methods: Another look at the jackknife. *The Annals of Statistics*, *7*(1), 1–26. https://doi.org/10.1214/aos/1176344552.

Fomby, T. B., Johnson, S. R., & Hill, R. C. (2012). *Advanced econometric methods*. *Advanced Econometric Methods*. Berlin, Germany: Springer. https://doi.org/10.1007/978-1-4419-8746-4.

Fornell, C. (1992). A national customer satisfaction barometer: The Swedish experience. *Journal of Marketing*, *56*(1), 6–21. https://doi.org/10.2307/1252129.

Fornell, C., Johnson, M. D., Anderson, E. W., Cha, J., & Bryant, B. E. (1996). The American customer satisfaction index: Nature, purpose, and findings. *Journal of Marketing*, *60*(4), 7. https://doi.org/10.2307/1251898.

Gifi, A. (1990). *Nonlinear multivariate analysis*. Retrieved from: Wiley. https://www.wiley.com/en-us/Nonlinear+Multivariate+Analysis-p-9780471926207.

Hair, J. F., Hult, G. T. M., Ringle, C. M., Sarstedt, M., & Thiele, K. O. (2017). Mirror, mirror on the wall: a comparative evaluation of composite-based structural equation modeling methods. *Journal of the Academy of Marketing Science*, *45*(5), 616–632. https://doi.org/10.1007/s11747-017-0517-x.

Hanafi, M. (2007). PLS path modelling: Computation of latent variables with the estimation mode B. *Computational Statistics*, *22*(2), 275–292. https://doi.org/10.1007/s00180-007-0042-3.

Hwang, H., Cho, G., Jung, K., Falk, C., Flake, J., Jin, M. J., & Lee, S. H. (in press). An approach to structural equation modeling with both factors and components: Integrated generalized structured component analysis. *Psychological Methods*.

Hwang, H., & Takane, Y. (2004). Generalized structured component analysis. *Psychometrika*, *69*(1), 81–99. https://doi.org/10.1007/BF02295841.

Hwang, H., & Takane, Y. (2014). *Generalized structured component analysis: A component-based approach to structural equation modeling*. New York, NY: Chapman and Hall/CRC Press.

Hwang, H., Takane, Y., & Jung, K. (2017). Generalized structured component analysis with uniqueness terms for accommodating measurement error. *Frontiers in Psychology*, *8*, 2137. https://doi.org/10.3389/fpsyg.2017.02137.

Hwang, H., Takane, Y., & Tenenhaus, A. (2015). An alternative estimation procedure for partial least squares path modeling. *Behaviormetrika*, *42*(1), 63–78. https://doi.org/10.2333/bhmk.42.63.

Johnson, M. D., Gustafsson, A., Andreassen, T. W., Lervik, L., & Cha, J. (2001). The evolution and future of national customer satisfaction index models. *Journal of Economic Psychology*, *22*(2), 217–245. https://doi.org/10.1016/S0167-4870(01)00030-7.

Jöreskog, K. G. (1970). Estimation and testing of simplex models. *British Journal of Mathematical and Statistical Psychology*, *23*(2), 121–145. https://doi.org/10.1111/j.2044-8317.1970.tb00439.x.

Jöreskog, K. G. (1978). Structural analysis of covariance and correlation matrices. *Psychometrika*, *43*(4), 443–477. https://doi.org/10.1007/BF02293808.

Jöreskog, K. G., & Wold, H. (1982). The ML and PLS techniques for modeling with latent variables: Historical and comparative aspects. In H. Wold & K. G. Jöreskog (Eds.), *Systems under indirect observation: Causality, structure, prediction, part I* (pp. 263–270). Amsterdam, Netherlands: North Holland.

Karatzoglou, A., Smola, A., Hornik, K., & Zeileis, A. (2004). kernlab - An S4 package for kernel methods in R. *Journal of Statistical Software*, *11*(9), 1–20. https://doi.org/10.18637/jss.v011.i09.

Krämer, N. (2007). *Analysis of high dimensional data with partial least squares and boosting*. Technische Universität Berlin, Fakultät IV - Elektrotechnik und Informatik: Technische Universität Berlin, Berlin. https://doi.org/10.14279/depositonce-1539.

Lohmöller, J. B. (1984). LVPLS Program Manual-Version 1.6. Zentralarchiv fur Empirische Sozialforschung. Koln: Universitat zu Koln.

Lohmö ller, J. B. (1989). *Latent variable path modeling with partial least squares*. New York, NY: Springer-Verlag. https://doi.org/10.1007/978-3-642-52512-4.

McDonald, R. P. (1996). Path analysis with composite variables. *Multivariate Behavioral Research*, *31*(2), 239–270. https://doi.org/10.1207/s15327906mbr3102_5.

Noonan, R., & Wold, H. (1982). PLS path modeling with indirectly observed variables: A comparison of alternative estimates for the latent variable. In K. G. Jöreskog & H. Wold (Eds.), *Systems under indirect observation: Causality, structure, prediction, part II* (pp. 75–94). Amsterdam, Netherlands: North Holland.

Rigdon, E. E. (2012). Rethinking partial least squares path modeling: In praise of simple methods. *Long Range Planning*, *45*(5–6), 341–358. https://doi.org/10.1016/j.lrp.2012.09.010.

Rigdon, E. E., Sarstedt, M., & Ringle, C. M. (2017). On comparing results from CB-SEM and PLS-SEM: Five perspectives and five recommendations. *Marketing ZFP*, *39*(3), 4–16. https://doi.org/10.15358/0344-1369-2017-3-4.

Ringle, C. M., Wende, S., & Will, A. (2005). SmartPLS 2.0 (beta). Hamburg, Germany: SmartPLS. Retrieved from http://www.smartpls.com

Rönkkö, M., McIntosh, C. N., Antonakis, J., & Edwards, J. R. (2016). Partial least squares path modeling: Time for some serious second thoughts. *Journal of Operations Management*, *47–48*, 9–27. https://doi.org/10.1016/j.jom.2016.05.002.

Sarstedt, M., Hair, J. F., Ringle, C. M., Thiele, K. O., & Gudergan, S. P. (2016). Estimation issues with PLS and CBSEM: Where the bias lies!. *Journal of Business Research*, *69*(10), 3998–4010. https://doi.org/10.1016/j.jbusres.2016.06.007.

ten Berge, J. M. F. (1993). *Least squares optimization in multivariate analysis*. Leiden, Netherlands: DSWO Press.

Tenenhaus, A., & Guillemot, V. (2017). RGCCA: Regularized and sparse generalized canonical correlation analysis for multiblock data. *R Package Version*, *2*(1), 2.

Tenenhaus, A., & Tenenhaus, M. (2011). Regularized generalized canonical correlation analysis. *Psychometrika*, *76*(2), 257. https://doi.org/10.1007/s11336-011-9206-8.

Tenenhaus, M. (2008). Component-based structural equation modelling. *Total Quality Management and Business Excellence*, *19*(7–8), 871–886. https://doi.org/10.1080/14783360802159543.

Tenenhaus, M., Tenenhaus, A., & Groenen, P. J. F. (2017). Regularized generalized canonical correlation analysis: A framework for sequential multiblock component methods. *Psychometrika*, *82*(3), 737–777. https://doi.org/10.1007/s11336-017-9573-x.

Vinzi, V. E., Trinchera, L., & Amato, S. (2010). PLS path modeling: From foundations to recent developments and open issues for model assessment and improvement. In V. Esposito Vinzi, W. W. Chin, J. Henseler, & H. Wang (Eds.), *Handbook of Partial Least Squares: Concepts, Methods and Applications* (pp. 47–82). Berlin, Heidelberg: Springer. https://doi.org/10.1007/978-3-540-32827-8_3

Wold, H. (1966). Estimation of principal components and related models by iterative least squares. In P. R. Krishnajah (Ed.), *Multivariate analysis* (pp. 391–420). NewYork, NY: Academic Press.

Wold, H. (1973). Nonlinear iterative partial least squares (NIPALS) Modelling: Some current developments. In P. R. Krishnaiah (Ed.), *Multivariate analysis-III* (pp. 383–407). New York, NY: Academic Press. https://doi.org/10.1016/B978-0-12-426653-7.50032-6.

Wold, H. (1982). Soft modeling: The basic design and some extensions. In K. G. Jöreskog & H. Wold (Eds.), *Systems under indirect observation: Causality, structure, prediction, part II* (pp. 1–54). Amsterdam, Netherlands: North Holland.

Wold, H. (1985). Partial least squares. In S. Kotz & N. L. Johnson (Eds.), *Encyclopedia of Statistical Sciences* (Vol. 6, pp. 581–591). New York, NY: Wiley.

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